Math 189 – Assignment #2 **Name:** Solution Set

Total: 30

1. State the form of the particular solution y_p for the following. Leave your answer with undetermined coefficients. (This means "Write down your initial guess for y_p but don't bother to solve for the constants.") Please note that the complementary solution for the homogeneous equation is $y_c = C_1 e^{2x} + C_2 e^{3x}$.

a) $y'' - 5y' + 6y = 4x^2 - 3$ $y_p = Ax^2 + Bx + C$

b) $y'' - 5y' + 6y = x^2 e^{-7x}$ $y_p = Ax^2 e^{-7x} + Bxe^{-7x} + Ce^{-7x}$

c) $y'' - 5y' + 6y = e^x \sin x$ $y_p = Ae^x \sin x + Be^x \cos x$

d) $y'' - 5y' + 6y = 2 + e^{3x}$ $y_p = A + Bxe^{3x}$

need extra x because this is the "bad case" - y_c already contains an e^{3x} term

(4)

2. Solve $y'' + 4y' = -4(y + 2x)$.

$y'' + 4y' + 4y = -8x$

y_c: $m^2 + 4m + 4 = 0$

$m = -2$ (repeated)

so $y_c = (C_1 + C_2 x)e^{-2x}$

y_p: $y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$y'' + 4y' + 4y = -8x$

$0 + 4A + 4Ax + 4B = -8x$

$4Ax + (4A + 4B) = -8x$

so this equals $-8x$; this equals zero

$4A = -8$

$A = -2$

$4A + 4B = 0$

$B = -A = 2$

$\therefore y_p = -2x + 2$

(3)

$y = y_c + y_p = (C_1 + C_2 x)e^{-2x} - 2x + 2$

3. Solve the following differential equation.

$y'' + 4y' - 21y = 5e^{2x}$ if when $x = 0$, $y = -\frac{5}{9}$ and $y' = -\frac{1}{9}$

(4)

y_c: $m^2 + 4m - 21 = 0$

$(m+7)(m-3) = 0$

$m = 3, -7$

$y_c = C_1 e^{3x} + C_2 e^{-7x}$

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y_p: $y_p = Ae^{2x}$

$y_p' = 2Ae^{2x}$

$y_p'' = 4Ae^{2x}$

∈ note: not "like" with any term in y_c so not "bad case"

$y'' + 4y' - 21y = 5e^{2x}$

$4Ae^{2x} + 8Ae^{2x} - 21Ae^{2x} = 5e^{2x}$

$(4A + 8A - 21A)e^{2x} = 5e^{2x}$

$-9A = 5$

$A = -5/9$

$\therefore y_p = -\frac{5}{9} e^{2x}$

$y = y_c + y_p = C_1 e^{3x} + C_2 e^{-7x} - \frac{5}{9} e^{2x}$

$y' = 3C_1 e^{3x} - 7C_2 e^{-7x} - \frac{10}{9} e^{2x}$

at $x = 0$,

$y = -5/9 = C_1 + C_2 - 5/9$

$0 = C_1 + C_2$

$C_1 = -C_2$

at $x = 0$,

$y' = -1/9 = 3C_1 - 7C_2 - 10/9$

$1 = 3C_1 - 7C_2$

$1 = 3C_1 + 7C_1$

$1 = 10C_1$

$C_1 = 1/10$

$C_2 = -1/10$

$y = \frac{1}{10} e^{3x} - \frac{1}{10} e^{-7x} - \frac{5}{9} e^{2x}$

4. A block of wood is floating in oil. A student pushes the block down 3 cm into the oil and holds it there at rest. Once the student lets go, the block then bobs up and down such that its equation of motion is $4\frac{d^2y}{dt^2}+4\frac{dy}{dt}+17y=0$, where y is the height of the block above its equilibrium position (in cm).

(5)

a) Find the height *y* as a function of the time elapsed after the student lets go.

b) Calculate the block's position after 1 seconds, 2 seconds, and 3 seconds have elapsed.

a) $4m^2+4m+17=0$

$m = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-4\pm\sqrt{16-16\cdot17}}{8}$

$= \frac{-4\pm16i}{8} = -\frac{1}{2}\pm 2i$

$\alpha = -\frac{1}{2}$ and $\beta = 2$

$y = e^{\alpha t}(C_1\cos\beta t + C_2\sin\beta t)$

$= e^{-t/2}(C_1\cos 2t + C_2\sin 2t)$

$\frac{dy}{dt} = -\frac{1}{2}e^{-t/2}(C_1\cos 2t + C_2\sin 2t) + e^{-t/2}(-2C_1\sin 2t + 2C_2\cos 2t)$

at $t=0$, $y=-3$

$-3 = e^0(C_1\cos 0 + C_2\sin 0)$

$-3 = C_1$

at $t=0$, $\frac{dy}{dt}=0$

$0 = -\frac{1}{2}C_1 + 2C_2$

$0 = \frac{3}{2} + 2C_2$

$C_2 = -\frac{3}{4}$

so $y = e^{-t/2}(-3\cos 2t - \frac{3}{4}\sin 2t)$

$= -3e^{-t/2}(\cos 2t + \frac{1}{4}\sin 2t)$

b) at $t=1$s, $y = -3e^{-1/2}(\cos 2 + \frac{1}{4}\sin 2) = 0.34358$ cm $= 0.3$ cm

$t=2$s, $y = -3e^{-1}(\cos 4 + \frac{1}{4}\sin 4) = 0.930195$ cm $= 0.9$ cm

$t=3$s, $y = -3e^{-3/2}(\cos 6 + \frac{1}{4}\sin 6) = -0.595969$ cm $= -0.6$ cm

make sure calculator is in rads

5. Consider the following variables.

   a) number of grains of sand in your shoes

   b) time spent by Math 254 students on computer games per week

   c) weight of the various chocolate bars in a vending machine

   d) brand names of chocolate bars in a vending machine

   Which of these are quantitative? a, b, c

   From the quantitive data, which are continuous? b, c

   2

6. The Gizmo Store is having a sale of its Bluetooth-enabled widgets which range in price from $25 to $75. Answer the following questions, being as specific as you can!

   a) If every widget is reduced in price by $10, what happens to the mean, median, range, and standard deviation of the widget prices?

   mean & median decrease by $10

   range & std dev stay the same

   2

   b) If, instead, the most expensive widget is reduced in price by $10, what happens to the mean, median, range, and standard deviation of the widget prices?

   mean - decreases
   median - will stay the same provided that it was originally below $65 otherwise, decreases
   range - decreases (by up to $10, depending on price of second-highest item)
   std dev - decreases

   2

   c) If, instead, every widget is reduced in price by 10%, what happens to the mean, median, range, and standard deviation of the widget prices?

   everything decreases by 10%

   1

7. A random sample of Technology students were asked how many hours they worked last week while on their co-op term, with the following results.

31, 42, 38, 26, 29, 29, 32, 35, 36, 37, 32, 30, 27, 43, 48, 30, 32, 33, 35, 39

The mean and standard deviation of this data are 34.2 and 5.7 hours/week, respectively. The histogram for this data is shown below.

a) Describe the shape and symmetry of the histogram.

unimodal and asymmetrical (skewed right) (1)

b) Find the percentage of measurements in the intervals $\bar{x} \pm s$, $\bar{x} \pm 2s$, $\bar{x} \pm 3s$ and fill out the following table. In your table, also state what percentages you expect to see in these intervals using either Tchebysheff or the Empirical Rule.

	interval	# of points	% of points	Empirical	Tchebysheff
$\bar{x} \pm s$	28.5 - 39.9	15	75%	~68%	≥0
$\bar{x} \pm 2s$	22.8 - 45.6	19	95%	~95%	≥ 75%
$\bar{x} \pm 3s$	17.1 - 51.3	all	100%	~99.7%	≥ 88.$\bar{8}$%

(2)

c) Do the percentages obtained in part b) agree with those given by the Empirical Rule? By Tchebysheff? Should they?

Tchebysheff - they should and they do (valid for all distributions)

Empirical - does not apply since not symmetrical but okay agreement except at $\bar{x} \pm 1s$

(1)

8. A set of data has a mean of 42 and a standard deviation of 8. What can you say about the proportion of measurements that lie above 58,

a) if you know absolutely nothing about the shape of the distribution?

by Tchebysheff, $\geq 75\%$ of the measurements i.e within 26 to 58,
so $\leq 25\%$ can lie above 58

(1)

b) if you know that the distribution is bimodal but perfectly symmetrical?

by Tchebysheff, $\leq 12.5\%$ can be above 58

(1)

c) if instead you know that the distribution is unimodal and beautifully symmetrical?

by the Empirical Rule
$\sim 2.5\%$ i.e above 58

(1)