Math 189 – Assignment #3 **Name:** Solution set

Total: 30

1. An individual is presented with three different glasses of soft drink, labeled A, B, and C. He is asked to taste all three and then list them in order of preference. Suppose that the same soft drink has actually been put into all three glasses. **Please show enough work** that I can tell which method you're using!

   a) How many simple events are there in this experiment? What probability would you assign to each one?

method 1: sample space

ABC BCA CBA
ACB BAC CAB

method 2: permutation

$_nP_r = {_3P_3} = 6$

so six simple events each with probability $1/6$

method 3: multiplication

$\underline{3}\ \underline{2}\ \underline{1}$

2

   b) What is the probability that A is ranked first?

method 1: 2 events in sample space so $2 \cdot 1/6 = 1/3$

method 2: $\frac{{_1P_1} \cdot {_2P_2}}{_3P_3} = \frac{2}{6} = \frac{1}{3}$

method 3: $\underline{A}\ \underline{2}\ \underline{1}$

so $P(A \text{ first}) = 2/6 = 1/3$

1

   c) What is the probability that either B or C is ranked first?

Sample space:

$P(B \text{ or } C \text{ first}) = \frac{4}{6} = 2/3$

since 4 events have B or C first

1

   d) Calculate your answer for c) again, but using a different method!

$P(B \text{ or } C \text{ first}) = 1 - P(A \text{ first})$
$= 1 - 1/3 = 2/3$

since P(A first) was calculated in part (b)

1

   e) Calculate your answer for c) a third time, but using a different method!

$P(B \text{ or } C \text{ first}) = P(B \text{ first}) + P(C \text{ first}) - P(B \text{ and } C \text{ first})$
$= 1/3 + 1/3 - 0 = 2/3$

1

   f) What is the probability that A is ranked first or B is ranked last?

$P(A \text{ first or } C \text{ last}) = P(A \text{ first}) + P(B \text{ last}) - P(A \text{ first and } B \text{ last})$
$=$ " $+$ " $- P(ACB)$
$= 1/3 + 1/3 - 1/6 = 1/2$

1

2. Students either from the Computing Systems Technology program or from the English department were asked who is the greatest fictional wizard ever, with the following results.

	Gandalf	Dumbledore
CST	77	63
English	33	27

2

$P(G) = \frac{n(G)}{n}$

$= \frac{110}{200} = \frac{11}{20} = 55\%$

Are "being a Computing Student" and "choosing Gandalf" independent? Explain your reasoning.

if C & G are independent, then

$P(C|G) = P(C)$ and $P(G|C) = P(G)$

$55\% = 55\%$ ✓

$\therefore$ yes, independent

$P(G|C) = \frac{n(CG)}{n(C)}$

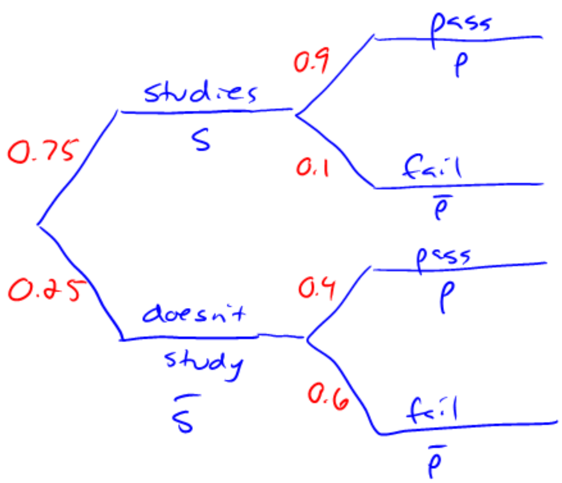
$= \frac{77}{140} = \frac{11}{20}$

$= 55\%$

3. A student in Pat's precalculus class studies for quizzes 75% of the time. If he studies, the probability of his passing is 90%. However, if he doesn't study, he'll likely only pass 40% of the time.

4

a) On the next test, what's the probability that this student will pass?
b) If the student fails, what's the probability that he didn't study?



$P(SP) = 0.675$

$P(S\bar{P}) = 0.075$

$P(\bar{S}P) = 0.1$

$P(\bar{S}\bar{P}) = 0.15$

a) $P(P) = P(SP) + P(\bar{S}P)$

$= 0.675 + 0.1$

$= 0.775$

$= 77.5\%$

b) $P(\bar{S} | \bar{P}) = \frac{P(\bar{S}\bar{P})}{P(\bar{P})}$

$= \frac{0.15}{1 - 0.775}$

$= 2/3 = 66.\overline{6}\%$

4. Consider a typical episode of the Mythbusters TV series.

   a) On Mythbusters, the team explores a myth (or urban legend) and at the end labels it either "Confirmed", "Plausible", or "Busted." Let's assume that the myths are randomly selected such that the labels given are all equally likely. If *x* is then the number of "Busted" myths resulting after twenty myths are examined, what probability distribution does *x* have? Explain briefly.

   binomial - fixed number of trials (20)
   - two outcomes: Busted / not
   - equal probability of Busted from trial to trial

   2

   b) Of the five Mythbusters, three of them have also worked on the special effects of "Star Wars: Episode 1". If you picked two of the five Mythbusters at random, let *x* be the number of Mythbusters chosen who have also worked on "Star Wars: Episode 1." What probability distribution does *x* have? Explain briefly.

   hypergeometric - fixed number of trials (2)
   - two outcomes: Star Wars / not
   - chosen without replacement so probabilities change from trial to trial

   2

5. On average, five people enter the intensive care unit (ICU) at a particular hospital on any one day.

   a) What probability distribution would best describe the variable *x*, where *x* is the number of people entering the ICU on a particular day?

   Poisson
   (because it's the number of events - discrete - in a given space or time)

   1

   b) Using your answer for (a), what is the probability that the number of people entering the ICU on a particular day is two? Less than or equal to two?

$$P(x=k) = \frac{\mu^k e^{-\mu}}{k!}$$

$$P(x=2) = \frac{5^2 e^{-5}}{2!} = 0.084 = \boxed{8.4\%}$$

$$P(x \le 2) = P(0) + P(1) + P(2) = 0.125 = \boxed{12.5\%}$$

2

6. You can insure a $50,000 diamond for its total value by paying a certain premium. If the probability of theft in a given year is estimated to be 0.5%, what premium should the insurance company charge if it wants the expected gain to equal $1000? Show your work.

let y = premium

4

x	$P(x)$
y	0.995 (no theft)
$y - 50000$	0.005 (theft)

$$E(x) = \sum x\,p(x)$$

$$1000 = 0.995y + 0.005(y - 50000)$$

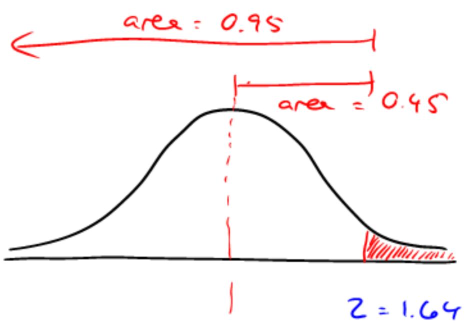
$$= 0.995y + 0.005y - 250$$

$$1250 = y$$

The company should charge $1250 for the premium

7. A publisher has discovered that the number of words contained in a manuscript by an experienced author are normally distributed, with a mean equal to the number specified in the author's contract and a standard deviation of 5,000 words. If the publisher wants to be almost certain (say, with a 95% probability) that the manuscript will have less than 75,000 words, what number of words should the publisher specify in the contract?

3



→ Corresponds to $z = 1.645$

$z = 1.645$

(1.64 and 1.65 also acceptable)

$$z = \frac{x - \mu}{\sigma}$$

$$\mu = x - z\sigma$$

$$= 75000 - (1.645)(5000)$$

$$= 66775$$

(66800 if used $z = 1.64$, 66750 if use $z = 1.65$)

The publisher should specify 67,000 words.

8. Suppose that some phenomenon has the following probability distribution.

$$f(x) = \begin{cases} \dfrac{k}{1+x^2} & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the following, giving exact answers.

a) Calculate k so that $f(x)$ is indeed a probability distribution function.

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 \frac{k\,dx}{1+x^2} = k \tan^{-1}x \Big|_0^1$$

$$1 = k \cdot \pi/4$$

so $k = 4/\pi$ (≈ 1.27324)

1

b) Calculate the probability of x being between $\dfrac{1}{\sqrt{3}}$ and 1.

$$P\left(\tfrac{1}{\sqrt{3}} < x < 1\right) = \int_{1/\sqrt{3}}^{1} \frac{4}{\pi(1+x^2)}\,dx$$

$$= \frac{4}{\pi} \tan^{-1} x \Big|_{1/\sqrt{3}}^{1}$$

$$= \frac{4}{\pi}\left(\pi/4 - \pi/6\right)$$

$$= 1/3$$

1

c) Calculate the mean value of x.

$$\mu = E(x) = \int_{-\infty}^{\infty} x\, f(x)\,dx$$

$$= \int_0^1 x \,\frac{4}{\pi}\, \frac{dx}{1+x^2}$$

let $u = 1+x^2$

$du = 2x\,dx$

$$= \frac{4}{\pi}\int_1^2 \frac{1}{2}\frac{du}{u}$$

$$= \frac{2 \ln|u|}{\pi} \Big|_1^2$$

$$\mu = \frac{2}{\pi}(\ln 2 - \cancel{\ln 1}^{\,0})$$

$$= \frac{2\ln 2}{\pi} \text{ or } \frac{\ln 4}{\pi}$$

(≈ 0.441271)

1