Section 5-16: Solutions

1 a) $1 = \int_{-\infty}^{\infty} f(x)\,dx$

$= \int_{6}^{12} kx^2\,dx$

$= \frac{1}{3} kx^3 \Big|_6^{12}$

$= \frac{1}{3}(1728 - 216)k$

$= 504k$

so $\boxed{k = \frac{1}{504}}$

b) $P(6 < x < 9) = \int_6^9 f(x)\,dx$

$= \int_6^9 \frac{1}{504} x^2\,dx$

$= \frac{1}{3(504)} x^3 \Big|_6^9$

$= \frac{171}{3(504)} = \frac{19}{56} \approx 0.339285$

$\approx \boxed{34\%}$

c) $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_6^{12} x\left(\frac{1}{504}x^2\right)dx$

$= \frac{1}{4(504)} x^4 \Big|_6^{12}$

$= \frac{19440}{4(504)} = \frac{135}{14} \approx 9.6428571 \boxed{\approx 9.6}$

2 a) $1 = \int_{-\infty}^{\infty} f(x)\,dx$

$1 = \int_1^2 \frac{k}{x}\,dx$

$1 = k \ln x \Big|_1^2$

$1 = k(\ln 2 - \cancel{\ln 1}^{\,0})$

$k = \frac{1}{\ln 2} \approx 1.44269$

≈ 1.4

b) $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_1^2 x \frac{k}{x}\,dx$

$= \int_1^2 k\,dx$

$= kx \Big|_1^2$

$= k = \frac{1}{\ln 2} \approx 1.4$

c) $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)^2\,dx - \mu^2$

$= \int_1^2 x^2 \frac{k}{x}\,dx - \mu^2$

$= \frac{kx^2}{2}\Big|_1^2 - \mu^2$

$= \frac{3}{2}k - \mu^2$

$= \frac{3}{2\ln 2} - \frac{1}{(\ln 2)^2} \approx 0.0826736$

so $\sigma \approx 0.287530$

≈ 0.29

3 a) $P(1 < x < 3) = \int_1^3 f(x)\,dx$

$$= \int_1^3 5e^{-5x}\,dx$$

$$= -e^{-5x}\Big|_1^3$$

$$= -e^{-15} + e^{-5}$$

$$\approx 0.00673764 \approx \boxed{0.67\%}$$

b) $P(x < 0.5) = \int_{-\infty}^{0.5} f(x)\,dx$

$$= \int_0^{0.5} 5e^{-5x}\,dx$$

$$= -e^{-5x}\Big|_0^{0.5}$$

$$= -e^{-2.5} + \cancel{e^0}^{\,1}$$

$$= 1 - e^{-2.5}$$

$$\approx 0.917915 \quad \boxed{\approx 92\%}$$

c) if you insist, you could do $\int_{0.5}^{\infty} f(x)\,dx$, but it's easier to say

$$P(x > 0.5) = 1 - P(x \leq 0.5)$$

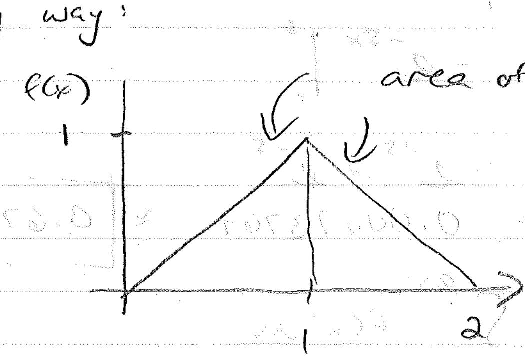
$$= 1 - (1 - e^{-2.5})$$

$$= e^{-2.5}$$

$$\approx 0.082 \quad \boxed{\approx 8.2\%}$$

4 $1 = \int_{-\infty}^{\infty} f(x)\,dx$

easy way:



area of each $\Delta = \frac{1}{2}$ so total is 1 ✓

slightly harder way:

$$1 = \int_0^1 x\,dx + \int_1^2 (2-x)\,dx$$

$$= \frac{x^2}{2}\Big|_0^1 + \left(2x - \frac{x^2}{2}\right)\Big|_1^2$$

$$= \frac{1}{2} + (4-2) - (2-\tfrac{1}{2})$$

$$= \frac{1}{2} + 2 - 1\tfrac{1}{2}$$

$$= 1$$ ✓