§ 31.1 #27

$$y = \frac{x+c}{\sec x + \tan x}$$

$$\frac{dy}{dx} = \frac{(\sec x + \tan x) - (x+c)(\sec x \tan x + \sec^2 x)}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x) - \sec x\,(x+c)(\tan x + \sec x)}{(\sec x + \tan x)^2}$$

$$= \frac{1 - \sec x\,(x+c)}{\sec x + \tan x}$$

DE: $\cos x \frac{dy}{dx} + \sin x = 1 - y$

$$\cos x \left(\frac{1 - \sec x\,(x+c)}{\sec x + \tan x} \right) + \sin x = 1 - \frac{x+c}{\sec x + \tan x}$$

$$\frac{\cos x - (x+c) + \sin x\,(\sec x + \tan x)}{\sec x + \tan x} = \frac{\sec x + \tan x - (x+c)}{\sec x + \tan x}$$

$$\frac{\cos x + \tan x + \frac{\sin^2 x}{\cos x} - (x+c)}{\sec x + \tan x} = \frac{\sec x + \tan x - (x+c)}{\sec x + \tan x}$$

but $\cos x + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

so $\frac{\sec x + \tan x - (x+c)}{\sec x + \tan x} = \frac{\sec x + \tan x - (x+c)}{\sec x + \tan x}$ ✓