Section 31.1: Solutions of Differential Equations

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differential equation (DE) ≡ an equation that contains derivatives or differentials

examples:

$$\frac{dy}{dx} = x^2 + 3$$

$$y\,dx = x\,dy$$

$$y'' + 3y' - 2y = x^2$$

jargon:

if the equation contains only first derivatives, it's called a first-order DE

if the equation contains second derivatives, it's called a second-order DE

∴ the order of the equation = order of the highest derivative in the equation

also, the degree of the equation = the highest power of the highest derivative

example: give the order and degree of the following DEs:

a) $\left(\frac{d^2y}{\quad}\right) + 4\left(\frac{dy}{\quad}\right)^2 = 7$

a) $\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dx}\right)^2 = 7$

$y'' + 4(y')^2 = 7$

order: 2nd

degree: 1st

note: $\frac{d^2y}{dx^2} = \left(\frac{d}{dx}\right)^2 y$

b) $\left(\frac{dy}{dx}\right)^3 = 7x^2$

order: 1st

degree: 3rd

note: this textbook emphasizes the degree of a DE, but most DE texts just look for whether y and all of its derivatives are raised to the first power (linear) or higher powers (non-linear)

solution to a DE ≡ a relation between variables that satisfies the DE

note: doesn't have to be a function

example: $\frac{dy}{dx} = x^2 + 5$) integrate!

example: $\frac{dy}{dx} = x^2 + 5$

integrate!

$y = \frac{x^3}{3} + 5x + C$

general solution - a solution to a DE that contains a number of arbitrary constants equal to the order of that DE

2nd order DE → 2 arbitrary constants