Section 31.2: Separation of Variables

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so we'll now learn some techniques for solving DEs

first up: separation of variables

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example: solve

$$dx\left(2\frac{dy}{dx}\right) = \frac{y(x+1)}{x}(dx)$$

$$2\,dy = \frac{y(x+1)}{x}\,dx$$

$$2\frac{dy}{y} = \frac{x+1}{x}\,dx$$

$$= \left(1+\frac{1}{x}\right)dx$$

⟸ variables are now separated

now integrate both sides:

$$2\ln y = x + \ln x + C$$

should really be $\ln|y|$ but your text isn't very strict about absolute values

can just put single constant on either side of equation

So, the solution to our DE is

$$\boxed{2 \ln y = x + \ln x + C}$$

note: this is a perfectly acceptable answer <u>even</u> <u>though</u> it's not solved for y

if it's specified to give an "explicit solution", that means you <u>do</u> have to solve for y (or the dependent variable)

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separation of variables:

consider a DE of first order and first degree:

$$\frac{dy}{dx} = f(x, y)$$

IF you can rewrite into the form

$$\underbrace{A(x)\,dx}_{\text{contains only } x} = \underbrace{B(y)\,dy}_{\text{contains only } y}$$

then can just integrate both sides to solve

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example: solve the following, giving an explicit solution.

$$\frac{y\,dt}{yt} + \frac{t\,dy}{yt} = \frac{0}{yt}$$

separated because each term contains single variable →

$$\frac{dt}{t} + \frac{dy}{y} = 0$$

$$\int \frac{dt}{t} + \int \frac{dy}{y} = \int 0$$

$$\ln t + \ln y = c$$

some constant

this is an implicit solution

$$\ln(ty) = c$$

$$ty = e^c$$

this is also just a constant

$$ty = c^*$$

$$\boxed{y = \frac{c^*}{t}}$$

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example: solve the following DE, given that $y = \pi/2$ when $x = 0$

$$\frac{2y\cos y\,dy - \sin y\,dy}{y\sin y} = \frac{y\sin y\,dx}{y\sin y}$$

$$2\cot y\,dy - \frac{dy}{y} = dx$$

$$\left(2\cot y - \frac{1}{y}\right)dy = dx$$

now integrate: $\displaystyle\int \left(2\cot y - \frac{1}{y}\right)dy = \int dx$

general solution $\rightarrow$ $2\ln|\sin y| - \ln y = x + C$

but when $x=0$, $y = \pi/2$

$$2\ln|\sin \pi/2| - \ln \pi/2 = 0 + C$$

$$2\cancel{\ln 1}^{\,0} \quad - \ln \pi/2 = C$$

$$-\ln \pi/2 = C$$

$$\boxed{2\ln|\sin y| - \ln y = x - \ln \pi/2}$$

perfectly acceptable answer $\nearrow$