Section 31.3: Integrating Combinations

Wednesday, April 08, 2015
2:52 PM

review of differentials:

$$dy = \frac{dy}{dx}\,dx$$

if $y = \sin x$, find dy:

$$dy = \cos x \, dx$$

if $y = x \sin x$, find dy:

$$dy = (x\cos x + \sin x)\,dx$$

if $z = x + y$, find dz:

$$dz = dx + dy$$

if $z = xy$, find dz:

$$dz = y\,dx + x\,dy$$

if $z = x^2 + y^2$, find dz:

$$dz = 2x\,dx + 2y\,dy$$

note: a different way of writing the same thing is:

$$d(x^2 + y^2) = 2x\,dx + 2y\,dy$$

examples of integrating combinations:

solve: $(2y + t)dy + y\,dt = 0$

$$\underbrace{2y\,dy}_{\text{only one variable}} + \underbrace{t\,dy + y\,dt}_{\text{this looks like a pattern we have up top}} = 0$$

if $z = ty$
$dz = t\,dy + y\,dt$

method #1: substitution

let $u = ty$
$du = t\,dy + y\,dt$

$2y\,dy + du = 0$ separated!

$\int 2y\,dy + \int du = \int 0$

$y^2 + u = c$

$$\boxed{y^2 + ty = c}$$

method #2: (direct)

$2y\,dy + t\,dy + y\,dt = 0$

$$2y\,dy + d(yt) = 0$$

now integrate

$$y^2 + yt = C$$