Section 31.5: Numerical Solutions of First-Order DEs

Tuesday, April 14, 2015
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note: in 8^{th} ed, this is Supplement 8

suppose you have some differential equation

$$\frac{dy}{dx} = f(x, y)$$

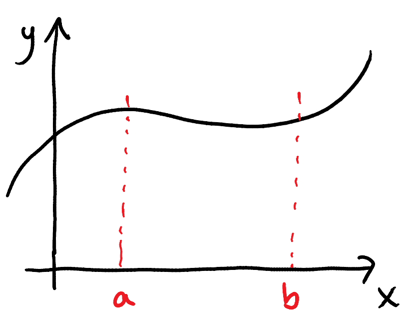
and you also know that when $\underbrace{x = x_0, \text{ that } y = y_0}_{\text{initial conditions}}$.

we'd like to solve for y and plug in the initial conditions to calculate values for our constant

→ but what if our usual methods (separation of variables, etc) fail?

numerical methods:

consider some curve:



the fundamental theorem of calculus says that

$$y(x=b) - y(x=a) = \int_a^b \frac{dy}{dx}\, dx$$

$$y(x=b) - y(x=a) = \int_a^b \frac{dy}{dx}\, dx$$

so $$y(b) = y(a) + \int_a^b \frac{dy}{dx}\, dx$$

but recall that our DE is

$$\frac{dy}{dx} = f(x,y)$$

and $$\boxed{y(b) = y(a) + \underbrace{\int_a^b f(x,y)\, dx}}$$

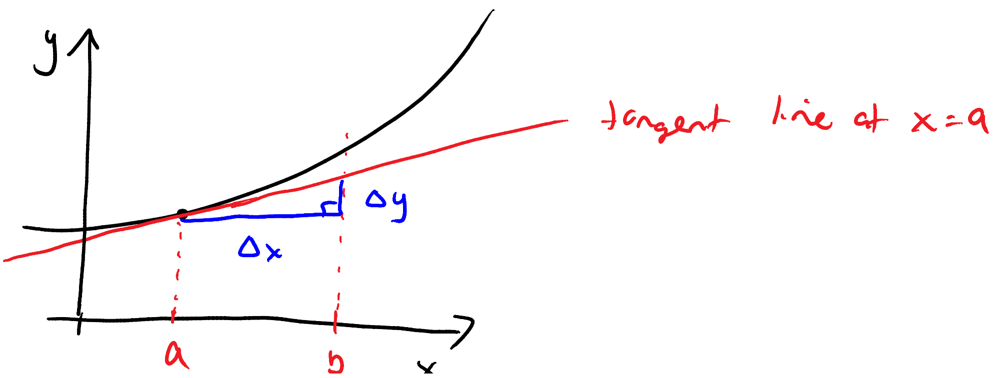
if we could integrate this directly, we'd be totally set, but what if we can't?

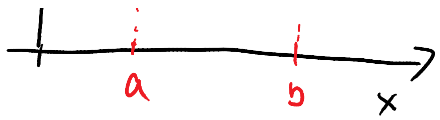
numerical methods use different approximations to find this integral

you've actually studied one method already:

→ linearization (Math 185)

linearization: (section 24.8)





$$y(b) = y(a) + \int_a^b \underbrace{\frac{dy}{dx}}_{} \, dx$$

assume slope is constant

so $m = y'(a)$

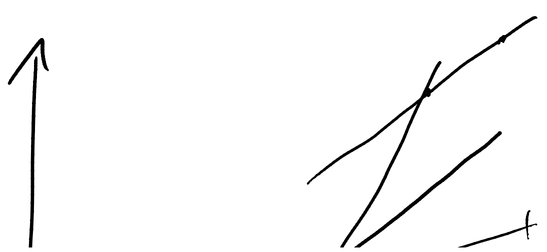
$$y(b) = y(a) + m(b-a)$$

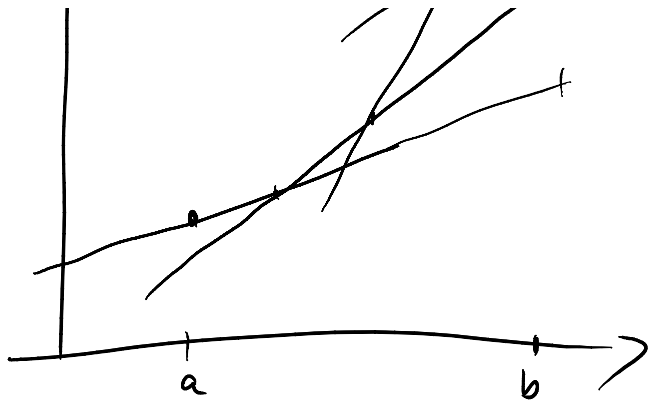
usually written: $\boxed{L(x) = y(a) + y'(a)(x-a)}$

works well provided that
a) x is close to a and/or
b) the curve is slowly varying

Euler's method: (assignment question)

- very similar to linearization
- the difference is that you do a <u>series</u> of linearizations, not just one

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