Section 31.6: Applications for first-order DEs

Wednesday, April 15, 2015
3:06 PM

(30.5 in 8th ed)

note: we will omit the electrical circuit applications

Why do we care about solving DEs? because they show up in so many applications!

example: If a population is allowed to grow unchecked (no predators, no disease, enough habitat & food), then the rate of growth for that population is proportional to the population at that time.

a) Write a differential equation that expresses this relationship. Use P for the population.

$$\frac{dP}{dt} \propto P$$

growth rate — population

$$\boxed{\frac{dP}{dt} = kP}$$

b) If the initial population is P_0 at $t = 0$, find an expression for $P(t)$.

$$\frac{dP}{} = kP$$

dt

$$\int \frac{dP}{P} = \int k\, dt$$

$$\ln|P| = kt + C$$

$$\ln P = kt + C$$

$$P = e^{kt+C}$$

$$= e^{kt}\, e^{C}$$

$$= C_1 e^{kt}$$

at $t = 0,\ P = P_0$ $\qquad P_0 = C_1 e^{0}$

$$C_1 = P_0$$

$$\boxed{P = P_0 e^{kt}}$$