Section 31.6: cont'd

Thursday, April 16, 2015
10:30 AM

handout, Question 1c:

doubling time: time it takes to double

$P = P_0 e^{kt}$

finding k: 5% → 100% in 2 years

$P = 0.05 P e^{k \cdot 2}$

$1 = 0.05 e^{2k}$

$20 = e^{2k}$

$\ln 20 = 2k$

$k = \frac{1}{2} \ln 20$

now find the doubling time

$P = P_0 e^{kt}$

$2\cancel{P_0} = \cancel{P_0} e^{kt}$

$2 = e^{kt}$

$\ln 2 = kt$

$t = \frac{\ln 2}{k} = \frac{\ln 2}{\frac{1}{2} \ln 20} \approx 0.4627$ years

≈ 0.46 years
≈ 0.5 years

Question #2:

let N = amount of radioactive materials

a) $$\frac{dN}{dt} = -kN$$

b) separable: $$\int \frac{dN}{N} = \int -k\,dt$$

$$\ln N = -kt + C$$

$$N = e^{-kt+C}$$

$$= e^{-kt}\,e^{C}$$ $= C_1$, some new constant

$$= C_1 e^{-kt}$$

at $t=0$, $N = N_0$

$$N_0 = C_1$$

$$\boxed{N = N_0 e^{-kt}}$$

c) half-life: time for half of the substance to decay away

finding k: halflife $t_{1/2} = 5.2$ days

$$N = N_0 e^{-kt}$$

$$\tfrac{1}{2}\cancel{N_0} = \cancel{N_0} e^{-k(5.2)}$$

$$\tfrac{1}{2} = e^{-5.2k}$$

$$\ln \tfrac{1}{2} = -5.2k$$

$$k = \frac{\ln \frac{1}{2}}{-5.2}$$

now find t: 90% decays away

$$N = N_0 e^{-kt}$$

$$\underbrace{0.1\,\cancel{N_0}}_{\text{how much is left}} = \cancel{N_0} e^{-kt}$$

$$0.1 = e^{-kt}$$

$$\ln 0.1 = -kt$$

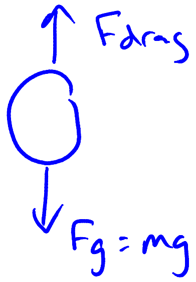
$$t = \frac{\ln 0.1}{-k} = \frac{\ln 0.1}{+(\ln \frac{1}{2})/+5.2}$$

$$\approx 17.274 \text{ days}$$

$$\approx 17 \text{ days}$$

Question 4: note: $F_{drag} \propto v$ only at low speeds
$\propto v^2$ at higher speeds

a) object in freefall with $F_{drag} \propto v$



$$F_{drag} = kv$$

↑ some constant

$$\sum \vec{F} = m\vec{a}$$

$$mg - kv = ma$$

$$\boxed{mg - kv = m\frac{dv}{dt}}$$

DE with m, k, and g are constants