Section 31.6: cont'd

Tuesday, April 21, 2015
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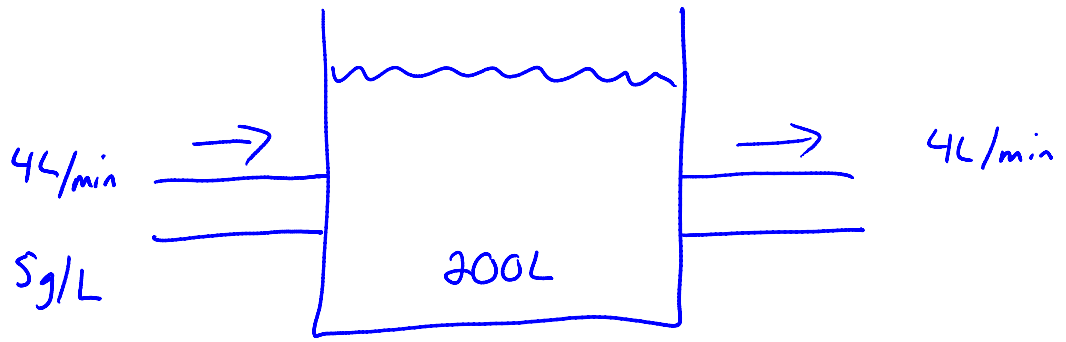
Question #5 from handout:



initially: 100g in 200L

m = mass of salt (g) $\rightarrow$ $\frac{m}{200}\ \frac{g}{L}$ is the concentration in the vat

incoming: rate of actual salt coming in

$$5\ g/L \cdot 4\ L/min = 20\ g/min$$

rate of actual salt going out is

$$\frac{m}{200}\ \frac{g}{L} \cdot 4\ L/min = \frac{m}{50}\ g/min$$

net rate of change of salt:

$$\frac{dm}{dt} = \left(\frac{dm}{dt}\right)_{incoming} - \left(\frac{dm}{dt}\right)_{outgoing}$$

$$= 20\ g/min - \frac{m}{50}\ g/min$$

50

writing without units:

$$\frac{dm}{dt} = 20 - \frac{m}{50}$$

$$\int \frac{dm}{20 - m/50} = \int dt$$

Let $u = 20 - m/50$

$$du = -\frac{1}{50}\, dm$$

$$-50\, du = dm$$

$$\int -50 \frac{du}{u} = \int dt$$

$$-50 \ln u = t + C_1$$

$$-50 \ln\left(20 - \frac{m}{50}\right) = t + C_1$$

$$\ln\left(20 - \frac{m}{50}\right) = -\frac{t}{50} \underbrace{-\frac{C_1}{50}}_{C_2}$$

$$20 - \frac{m}{50} = e^{-t/50 + C_2}$$

$$= e^{-t/50} \underbrace{e^{C_2}}_{C_3}$$

$$-t/50$$

$$20 - \frac{m}{50} = C_3 e^{-t/50}$$

$$-\frac{m}{50} = C_3 e^{-t/50} - 20$$

$$m = -50 C_3 e^{-t/50} + 1000 \qquad (C_4 = -50 C_3)$$

$$m = C_4 e^{-t/50} + 1000$$

initial condition: at $t = 0$, $m = 100$ g

$$100 = C_4 e^{0} + 1000$$

$$C_4 = -900$$

$$\boxed{m = 1000 - 900 e^{-t/50}}$$

note: DE is $\frac{dm}{dt} = 20 - \frac{m}{50}$

$$\frac{dm}{dt} + \frac{m}{50} = 20$$

linear with $P(t) = \frac{1}{50}$

integrating factor: $e^{\int P(t)\,dt} = e^{\int \frac{1}{50}\,dt}$

$= e^{t/50}$

b) $\lim_{t\to\infty} m = \lim_{t\to\infty} \left(1000 - 900e^{-t/50}\right)$

$900e^{-t/50}$ approaches zero

$= 1000\ g$

note: concentration of salt in tank as $t\to\infty$ is $\frac{1000\ g}{200\ L} = 5\ \frac{g}{L}$, same as incoming brine

c) at what time does $m = 600$?

$$m = 1000 - 900e^{-t/50}$$

$$600 = 1000 - 900e^{-t/50}$$

$$-400 = -900e^{-t/50}$$

$$\frac{4}{9} = e^{-t/50}$$

$$-50 \ln \frac{4}{9} = t$$

$$t \approx 40.5465 \text{ min}$$

≈ 40 min
≈ 41 min
≈ 40.5 min
} all acceptable