

# Section 31.9: cont'd

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3:07 PM

solve:  $2y'' + 5y' - 3y = e^x + 4e^{2x}$

Complementary sol'n:

aux eqn:  $2m^2 + 5m - 3 = 0$

ac = -6  
-1 6

$$2m^2 + 6m - m - 3 = 0$$

$$2m(m+3) - (m+3) = 0$$

$$(2m-1)(m+3) = 0$$

$$m = \frac{1}{2}, -3$$

$$y_c = C_1 e^{-3x} + C_2 e^{\frac{1}{2}x}$$

particular solution:

RHS:  $e^x + 4e^{2x}$

$$y_p = Ae^x + Be^{2x}$$

$$y_p' = Ae^x + 2Be^{2x}$$

$$y_p'' = Ae^x + 4Be^{2x}$$

OE:  $2y'' + 5y' - 3y = e^x + 4e^{2x}$

$$2Ae^x + 8Be^{2x} + 5Ae^x + 10Be^{2x} - 3Ae^x - 3Be^{2x} = e^x + 4e^{2x}$$

$$4Ae^x + 10Be^{2x} - 3Be^{2x} = e^x + 4e^{2x}$$

$$4Ae^x + 15Be^{2x} = e^x + 4e^{2x}$$

So  $4A = 1$  and  $15B = 4$   
 $A = 1/4$  and  $B = 4/15$

$$y_p = \frac{1}{4}e^x + \frac{4}{15}e^{2x}$$

$$y = y_c + y_p$$

$$= C_1 e^{-3x} + C_2 e^{1/2x} + \frac{1}{4}e^x + \frac{4}{15}e^{2x}$$

the "bad case":

example: solve:

$$y'' - 5y' + 4y = 10e^x$$

complementary soln:

aux eqn:  $m^2 - 5m + 4 = 0$

$$(m - 4)(m - 1) = 0$$

$$m = 1, 4$$

$$y_c = C_1 e^x + C_2 e^{4x}$$

particular soln:

aux eqn:

particular sol'n:

$$\text{RHS} = 12e^x$$

$$y_p = \cancel{Ae^x}$$

← "bad case" because  $Ae^x$  is a like term to one of the terms in  $y_c$

$$y_p = Ax e^x$$

← so we fix this by multiplying by  $x$  until terms aren't "like" any more

now substitute back into DE:

$$\begin{aligned} y_p &= Ax e^x \\ y_p' &= \downarrow \quad \searrow \quad \rightarrow \quad Ae^x \\ & Ax e^x + Ae^x \\ y_p'' &= \downarrow \quad \searrow \quad \rightarrow \quad Ae^x \quad \rightarrow \quad Ae^x \\ & Ax e^x + Ae^x + Ae^x \\ &= Ax e^x + 2Ae^x \end{aligned}$$

$$y'' - 5y' + 4y = 12e^x$$

$$Ax e^x + 2Ae^x - 5Ax e^x - 5Ae^x + 4Ax e^x = 12e^x$$

$$-3Ae^x = 12e^x$$

$$-3A = 12$$

$$A = -4$$

$$y_p = -4x e^x$$

full solution:

$$y = y_c + y_p$$

$$y = C_1 e^x + C_2 e^{4x} - 4x e^x$$

example: solve:

$$y'' + 9y = 4 \sin 3x$$

complementary solution:

$$m^2 + 9 = 0$$

$$m^2 = -9$$

$$m = \pm 3i = \alpha \pm \beta i$$

$$y_c = e^{\alpha x} (C_1 \sin \beta x + C_2 \cos \beta x)$$

$$= C_1 \sin 3x + C_2 \cos 3x$$

particular solution

$$RHS = 4 \sin 3x$$

"bad case"