# Section 31.9: cont'd

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1:48 PM

recall from last time:

solve $y'' + 9y = 4\sin 3x$

we found $y_c = C_1 \sin 3x + C_2 \cos 3x$

particular sol'n: RHS: $4\sin 3x$

THE BAD CASE!

$y_p = A\cancel{\sin 3x} + B\cancel{\cos} 3x$

$= x(A\sin 3x + B\cos 3x)$

$= Ax\sin 3x + Bx\cos 3x$

$y_p = x(A\sin 3x + B\cos 3x)$

$y_p' = A\sin 3x + B\cos 3x + \underbrace{x(3A\cos 3x - 3B\sin 3x)}$

$y_p'' = 3A\cos 3x - 3B\sin 3x + 3A\cos 3x - 3B\sin 3x$

$+ x(-9A\sin 3x - 9B\cos 3x)$

$= 6A\cos 3x - 6B\sin 3x - 9x(A\sin 3x + B\cos 3x)$

now plug back into DE:

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$$y'' + 9y = 4\sin 3x$$

$$\left[6A\cos 3x - 6B\sin 3x - 9x(A\sin \cancel{3x + B\cos} 3x)\right]$$

$$+ 9x(A\sin \cancel{3x + B\cos} 3x) = 4\sin 3x$$

$$6A\cos 3x - 6B\sin 3x = 4\sin 3x$$

so $6A = 0$ and $-6B = 4$

$A = 0$ $B = -2/3$

$$y_p = -\frac{2}{3}x\cos 3x$$

full solution: $y = y_c + y_p$

$$\boxed{y = C_1\cos 3x + C_2\sin 3x - 2/3\, x\cos 3x}$$

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example: solve

$$y'' - 2y' + y = xe^{2x} - e^{2x}$$

given that $y' = 4$ and $y = -2$ when $x = 0$.

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complementary sol'n: $m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1$

$y_c = (c_1 + c_2 x)e^x$

particular sol'n: $\quad$ RHS $= xe^{2x} - e^{2x}$

$y_p = Axe^{2x} + Be^{2x}$

$y_p' = 2Axe^{2x} + Ae^{2x} + 2Be^{2x}$

$y_p'' = 4Axe^{2x} + 2Ae^{2x} + 2Ae^{2x} + 4Be^{2x}$

$= 4Axe^{2x} + 4Ae^{2x} + 4Be^{2x}$

plug back in:

$y'' - 2y' + y = xe^{2x} - e^{2x}$

$(\cancel{4Axe^{2x}} + 4Ae^{2x} + \cancel{4Be^{2x}}) - \cancel{4Axe^{2x}} - 2Ae^{2x} - \cancel{4Be^{2x}}$

$+ Axe^{2x} + Be^{2x} = xe^{2x} - e^{2x}$

$Axe^{2x} + 2Ae^{2x} + Be^{2x} = xe^{2x} - e^{2x}$

$Axe^{2x} + (2A+B)e^{2x} = xe^{2x} - e^{2x}$

$A = 1$ and $2A + B = -1$

$B = -3$

$$y_p = Axe^{2x} + Be^{2x}$$

$$\boxed{y_p = xe^{2x} - 3e^{2x}}$$

$$y = y_c + y_p$$

$$= (c_1 + c_2 x)e^{x} + xe^{2x} - 3e^{2x}$$

so, initial conditions : when $x=0$, $y=-2$

$$-2 = (c_1 + 0) + 0 - 3$$

$$c_1 = 1$$

when $x=0$, $y' = 4$:

$$y' = (c_1 + c_2 x)e^{x} + c_2 e^{x} + 2xe^{2x} + e^{2x}$$
$$- 6e^{2x}$$

$$4 = c_1 + c_2 + 0 + 1 - 6$$

$$4 = 1 + c_2 + 0 + 1 - 6$$

$$c_2 = 8$$

$$y = (1+8x)e^{x} + (x-3)e^{2x}$$