Section 22: Useful Counting Rules

Wednesday, May 20, 2015
2:08 PM

the addition rule:

How many integers from 1 to 20 inclusive are

a) evenly divisible by 2?
b) " " " 3?
c) " " " 2 or 3?

a) divisible by 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

so 10 integers

b) divisible by 3: 3, 6, 9, 12, 15, 18

so 6 integers

c) divisible by 2 or 3: 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

so (13) integers

why not 16? because there's overlap in the two list (3 entries overlap)

$$n(A \text{ or } B) = n(A) + n(B) - n(AB)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

note: if $P(AB) = 0$, then events A & B are mutually exclusive

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multiplication rule:

consider an experiment that is performed in k steps and all of the steps are independent (the result of step 1 does not influence the outcome of step 2)

then the total number of possible outcomes for the experiment is:

$$n_{TOT} = n_1 \cdot n_2 \cdot n_3 \cdot \ldots \quad n_k$$

↑ the number of outcomes for step k

example:

What's the number of 4-digit PINs
a) in total?
b) that start with a 9?
c) that end in a 4?
d) that start with a 9 or end in a 4?

a) $\_\_\_ \_\_\_ \_\_\_ \_\_\_$ $= 10^4$
10 10 10 10

b) $\_\_\_ \_\_\_ \_\_\_ \_\_\_$ $= 10^3$
1 10 10 10

c) same as b, 10^3

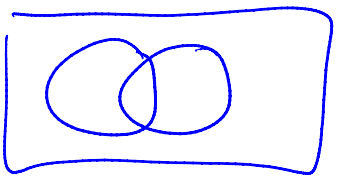
d) n(start with 9 or end in 4)

$= n(\text{start } 9) + n(\text{end } 4) - (\text{both})$

(both): $\_1\_ \_10\_ \_10\_ \_1\_ = 100$

$= 1000 + 1000 - 100$

$= 1900$



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example: what is the total number of BC licence plates for cars (ignoring reserved words and personalized plates)?

three patterns:

letter	letter	letter	number	number	number
number	number	number	letter	letter	letter
letter	number	number	number	letter	letter

top pattern: $\underset{26}{\_\_}\ \underset{26}{\_\_}\ \underset{26}{\_\_}\ \underset{10}{\_\_}\ \underset{10}{\_\_}\ \underset{10}{\_\_}$ $= 26^3 10^3$

$= 17\ 576\ 000$

so total number of patterns: $3(17\ 576\ 000)$

$= 52\ 728\ 000$

example: In the mythical Canadian province of Gondor, licence plates are issued with the pattern letter - letter - letter - number - number. Due to recent political events, the combination EYE is no longer allowed. How many legal licence plates are there?