Section 2.5: Combinations and Permutations

Wednesday, May 27, 2015
2:12 PM

factorials: $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$1! = 1$

$0! = 1$ ← okay, this one's a little weird

example: How many 4-digit PINs are there if

a) repetition of digits is allowed?

b) " " " is not allowed?

a) $\underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} = 10^4$

b) $\underline{10} \quad \underline{9} \quad \underline{8} \quad \underline{7} = 5040$

note: $10 \cdot 9 \cdot 8 \cdot 7 = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$= \dfrac{10!}{6!}$

permutation: an ordered arrangement of r objects chosen without replacement from n possibilities

notation: nPr ← I will use

P_r^n

$P(n, r)$

$nPr = \dfrac{n!}{(n-r)!}$

example: How many 5-digit case-sensitive alphanumeric

passwords are there if repetition is not allowed?

$${}_{62}P_5 = 776\ 520\ 240$$

↑

(26 letters × 2) + 10

↑ upper / lower case ↑ digits

---

combination – an unordered arrangement of r objects chosen without repetition from n possibilities

notation: nC_r ⇐ I will use

C_r^n

$C(n,r)$

$\binom{n}{r}$

$$nC_r = \frac{n!}{r!\,(n-r)!}$$

[why? $nC_r = \frac{nP_r}{r!}$]

↑

$r!$ = number of ways to order r objects

example: Pat is ordering pizza from Pongo, she has the option of 3 toppings chosen from a list of 15. How many different pizzas could she potentially order?

$${}_{15}C_3 = 455$$

If, in addition to the toppings, Pat has a

choice of three different pizza crusts, how many pizzas could she potentially order now?

$${}_{15}C_3 \; {}_3C_1 = 1365$$

example: In the BC 6/49 lottery, the customer chooses without replacement six numbers from 49 possibilities. These numbers are then compared with the six numbers chosen at random at the end of the week.

How many different tickets could you choose that end up with 4 winning numbers?

$${}_6C_4 \; {}_{43}C_2 = 15 \cdot 903$$
$$= 13545$$

${}_6C_4$: choosing 4 winning numbers out of 6

${}_{43}C_2$: choosing 2 losing numbers

How many different tickets in total are there?

$${}_{49}C_6 = 13\,983\,816$$