Section 3.2: cont'd

Friday, May 29, 2015
8:35 AM

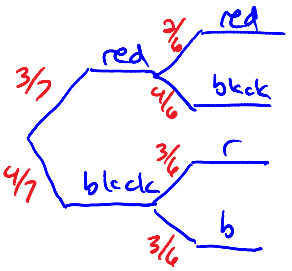
consider the following experiment:

1. place 3 red and 4 black marbles in a bag
2. mix thoroughly
3. draw one marble and record whether it is red or black
4. repeat step 3 for a total of four trials

under what conditions is this a binomial experiment?

→ if you replace the marble you drew between trials

otherwise the probabilities change:



without replacement

↑ not same probabilities

note: if there were 30 million red and 40 million black balls, the probabilities of red vs black would change so little without replacement that the experiment is approximately binomial

so there's a rule for selection without replacement:

if $\frac{n}{N} \geq 0.05$,

n = number of trials
N = total number

can't approximate with binomial

in other words, if

population ≥ 20 × sample size,

then can use binomial

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how do we calculate the binomial probability distribution?

suppose we have 3 trials and the probability of success $P(S) = p$ and the probability of failure is $P(F) = q$ where $q = 1 - p$

sample space:

	# successes	probability of event
SSS	3	$p \cdot p \cdot p$
SSF	2	$p \cdot p \cdot q$
SFS	2	$p \cdot q \cdot p$
SFF	1	$p \cdot q \cdot q$
FSS	2	
FSF	1	
FFS	1	
FFF	0	

1 way to get 3 successes: $P(3) = 1 \cdot p^3$

3 " " " 2 " $P(2) = 3 \cdot p^2 q$

3 " " " 1 " $P(1) = 3 \, p \, q^2$

1 " " " 0 " $P(0) = 1 \, q^3$

generalizing:

for binomial: $P(x = k) = {}_nC_k \; p^k \; q^{n-k}$

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for binomial quiz: 10 questions $P(S) = p = 1/3$ each

$x = 0$ $P(0) = {}_{10}C_0 \; (1/3)^0 (2/3)^{10} = 0.017$

$x = 0$ $\quad p(0) = {}_{10}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{10} = 0.017$

$p(1) = {}_{10}C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^9 = 0.086$

x	$p(x)$
0	0.017
1	0.086
2	0.195
3	0.260
4	0.227
5	0.136
6	0.056
7	0.016
8	0.003
9	0.000 33
10	0.000 0169