Section 4.1: cont'd

Wednesday, June 03, 2015
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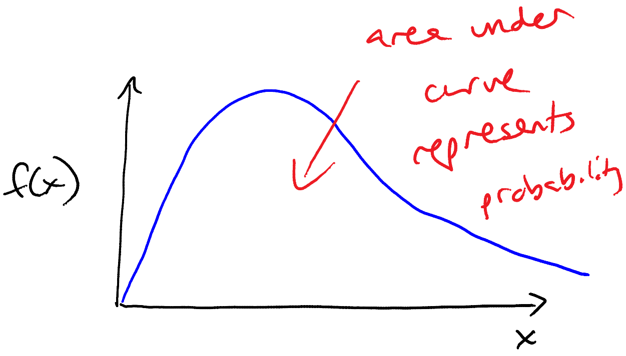
continuous probability distributions:



properties:

- the total area under the curve is equal to 1

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

- the probability that x falls between values a and b is

$$P(a < x < b) = \int_a^b f(x)\,dx$$

note: $P(x = a) = 0$

$\therefore$ $P(x \geq a) = P(x > a)$ if x is continuous

note: not true if x is discrete

so, what $f(x)$ do you use? which function do you pick?

answer: the one that best models the situation (if such a model exists)

$\underline{\sigma}$

the one that best fits your data

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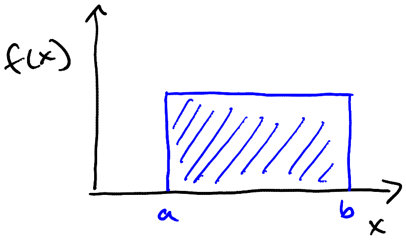
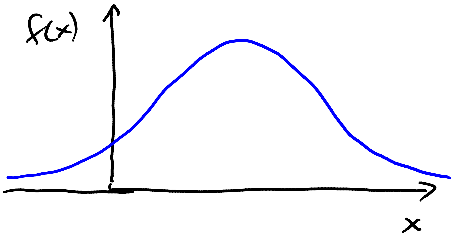
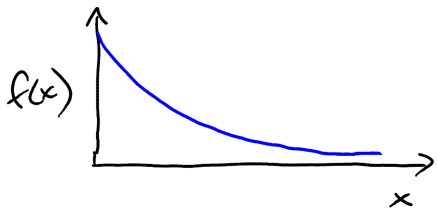
examples of continuous distributions :



uniform distribution



normal distribution
(Bell curve)



exponential distribution

(we're not going to do this one in any detail)

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so, how do you find the mean? (mean = average value)

$$\mu = \int_{-\infty}^{\infty} x \, f(x) \, dx$$

discrete:

$$\mu = \Sigma \, x \, p(x)$$

and the variance?

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 \, f(x) \, dx \; - \mu^2$$

discrete:

$$\sigma^2 = \Sigma x^2 p(x) - \mu^2$$

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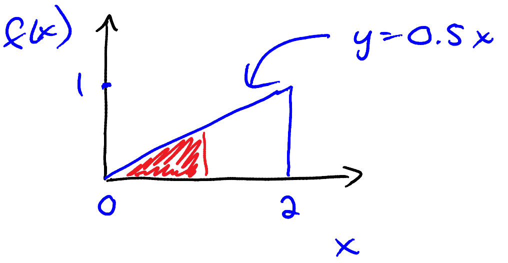
example: Let x denote the amount of time for which a book is checked out of the library if it is on two-hour reserve.

Suppose that

$$f(x) = \begin{cases} 0.5x & \text{for } 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

a) calculate the probability that $x \le 1$ hour



method #1

$$P(0 \le x \le 1) = \text{shaded area}$$
$$= \tfrac{1}{2} bh$$
$$= \tfrac{1}{2}(1)(\tfrac{1}{2})$$
$$= \tfrac{1}{4}$$

but if you insist,

method #2:

$$P(0 \le x \le 1) = \int_{-\infty}^{\infty} f(x)\,dx$$

$$= \int_0^1 0.5x\,dx$$

$$= \frac{x^2}{4}\Big|_0^1 = \frac{(1)^2}{4} - \frac{(0)^2}{4} = \frac{1}{4} \text{ or } 25\%$$

b) calculate the mean value of x and its standard deviation

$$\mu = \int_{-\infty}^{\infty} x \, f(x) \, dx$$

$$= \int_0^2 x \, (0.5 x) \, dx$$

$$= \int_0^2 \frac{1}{2} x^2 \, dx$$

$$= \frac{1}{6} x^3 \Big|_0^2 = \frac{8}{6} = \frac{4}{3}$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) \, dx - \mu^2$$

$$= \int_0^2 x^2 \, (0.5 x) \, dx - \mu^2$$

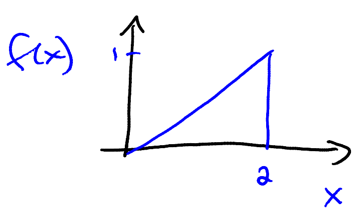
$$= \int_0^2 \frac{1}{2} x^3 \, dx - \mu^2$$

$$= \frac{1}{8} x^4 \Big|_0^2 - \mu^2$$

$$= 2 - \left(\frac{4}{3}\right)^2 = 2 - \frac{16}{9} = \frac{2}{9}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3} \approx 0.47$$

c) verify that the area under the curve is indeed equal to one



$$\text{area} = \tfrac{1}{2} b h = \tfrac{1}{2}(2)(1) = 1 \quad \checkmark$$