Section 4.2: The Uniform Probability Distribution

Wednesday, June 03, 2015
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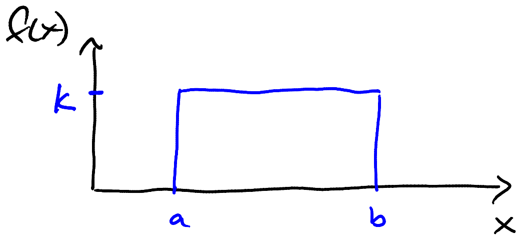

$k =$ a constant

so, what's the value of k for $f(x)$ to truly be a probability density function?

$$\text{area} = 1$$
$$l\,w = 1$$
$$k(b-a) = 1$$
$$k = \frac{1}{b-a}$$

so continuous uniform case has

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for} \quad a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

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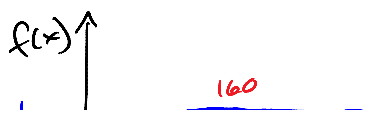
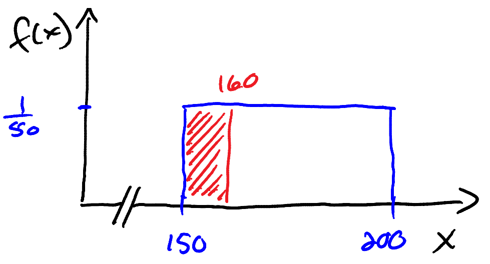
example: Suppose the research department of a steel manufacturing company believes that one of the company's rolling machines is producing sheets of steel of varying thickness. The thickness is a uniform random variable with values between 150 and 200 mm. Any sheets produced less than 160 mm thick must be scrapped because they are unacceptable to buyers.

a) Calculate the fraction of steel sheets produced by this machine that have to be scrapped.



$P(x < 160) = 1/5$



$P(x < 160) = 1/5$

(area between 150 & 160)

b) Calculate the mean and standard deviation of the thickness of sheets. What is the probability that a randomly selected sheet will lie within 1 std deviation of the mean? within 2 std devs?

$\mu = 175$ mm (halfway between 150 & 200)

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

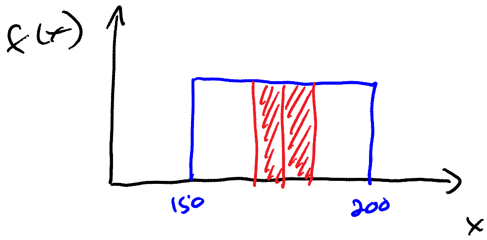
$$= \int_{150}^{200} x^2 \cdot \frac{1}{50}\,dx - 175^2$$

$$= \frac{x^3}{150}\Big|_{150}^{200} - 175^2$$

$$\approx 208.\bar{3}$$

$\sigma \approx 14.4$ mm
≈ 14 mm

$\mu \pm 1\sigma = 175 \pm 14$ mm $= 161$ to 189 mm



$P(161 < x < 189) = \frac{1}{50}(2 \cdot 14)$

$= 56\%$

$\mu \pm 2\sigma = 175 \pm 28$ mm $= 147$ mm to 203 mm

$P(147 < x < 203) = 100\%$