Section 8.2: cont'd

Tuesday, June 09, 2015
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Assignment #4 due on Tuesday, June 16^{th}

Central Limit Theorem:

if random samples of n observations are drawn from a non-normal population with finite mean μ and standard deviation σ,

when n is large ($n \geq 30$) the distribution of the sample mean $\bar{x}$ is approximately normally distributed with

mean μ

standard deviation $\frac{\sigma}{\sqrt{n}}$

and this approximation gets more accurate as n increases

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example: suppose that at a large university, there are very large class sizes for introductory calculus

- consider the probability of an individual getting >90% on a test

is this probability for an individual the same or different than the probability that the class average is > 90% ?

- different! it's far more likely for an individual to get 90%

than for the average on an entire group to be ≥ 90%

and the larger the group, the less likely it is.

what the central limit theorem says for this example is that:

if on the test the average for an individual is 72 out of 100 with a standard deviation of 8 out of 100

but the class average for a section of 36 students will still be 72/100 but the std dev for that group will now be $\frac{8}{\sqrt{n}} = \frac{8}{\sqrt{36}} = \frac{8}{6} = 1.\bar{3}$

Summary: if $n \geq 30$, the mean of a group will be approx normally distributed with

mean μ
std dev $\frac{\sigma}{\sqrt{n}}$

the sum of a group has

$$\left.\begin{array}{l} \text{mean } n\mu \\ \text{std dev } \sigma\sqrt{n} \end{array}\right\} \text{prev results multiplied by } n$$

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example: The weight of luggage checked by airline passengers is a random variable with a mean of 50 lbs and a standard deviation of 30 lbs. The total baggage limit for 100 randomly selected passengers is 5750 lbs.

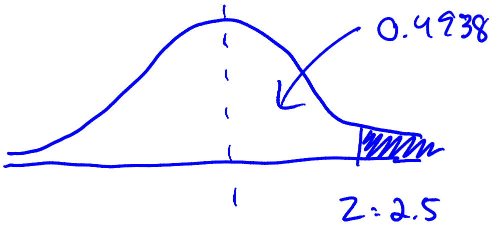
What is the approximate probability that the baggage limit will be exceeded?

because $n \geq 30$, the sum of the weights of luggage will be normally distributed

$$\mu_{\text{total weight of luggage}} = n\,\mu_{\text{individual luggage}} = 100 \cdot 50 = 5000$$

$$\sigma_{\text{total weight}} = \sigma\sqrt{n} = 30\sqrt{100} = 300$$

$$Z = \frac{X-\mu}{\sigma} = \frac{5750 - 5000}{300} = 2.5$$



$$P(Z > 2.5) = 0.5 - 0.4938 = 0.0062 = 0.62\,\%$$