Review:

Thursday, June 18, 2015
10:42 AM

Solve: $x\,dy + 2y\,dx = 3x\,dx$

Give an explicit solution.

$$x\frac{dy}{dx} + 2y = 3x$$

$$\frac{dy}{dx} + \frac{2}{x}y = 3$$

integrating factor $e^{\int P(x)dx} = e^{\int \frac{2}{x}dx}$

$= e^{2\ln x}$

$= e^{\ln x^2}$

$= x^2$

$$x^2\frac{dy}{dx} + 2xy = 3x^2$$

$$\frac{d}{dx}(x^2y) = 3x^2$$

$$\int d(x^2y) = \int 3x^2\,dx$$

now integrate:

$$x^2y = x^3 + C$$

$$\boxed{y = x + \frac{C}{x^2}}$$

Solve:

$$y'' - 6y' + 8y = 24x^3$$

Complementary solution:

aux eqn: $m^2 - 6m + 8 = 0$

$(m-2)(m-4) = 0$

$m = 2, 4$

$$y_c = C_1 e^{2x} + C_2 e^{4x}$$

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particular solution:

$$y_p = Ax^2 + Bx + C$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

sub back into DE:

$$y'' - 6y' + 8y = 24x^2$$

$$2A - 6(2Ax + B) + 8(Ax^2 + Bx + C) = 24x^2$$

$$2A - 12Ax - 6B + 8Ax^2 + 8Bx + 8C = 24x^2$$

$$8Ax^2 + x(-12A + 8B) + (2A - 6B + 8C) = 24x^2 + 0x + 0$$

so $8A = 24$ and $A = 3$

and $-12A + 8B = 0$

$-36 + 8B = 0$

$B = 36/8 = 9/2$

and $2A - 6B + 8C = 0$

$6 - 27 + 8C = 0$

$8C = 21$

$C = 21/8$

$$y_p = 3x^2 + \frac{9}{2}x + \frac{21}{8}$$

$$y = y_c + y_p = \boxed{C_1 e^{2x} + C_2 e^{4x} + 3x^2 + \frac{9}{2}x + \frac{21}{8}}$$