Section 31.1: Solutions of Differential Equations

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11:38 AM

differential equation (DE) ≡ an equation that contains derivatives or differentials

$$\frac{dy}{dx} = x^2 + 3 \qquad y'' + 3y' - 2y = x^2$$

jargon:

if the equation contains only first derivatives, it's called a first-order DE

if the equation contains second derivatives, it's called a second-order DE

∴ the order of the equation = order of the highest derivative in the equation

the degree of the equation = the highest power of the highest derivative

example: give the order and degree of the following DEs:

a) $\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dx}\right)^2 = 7$

$y'' + 4(y')^2 = 7$ same eqn, different notation

2nd order
1st degree

b) $4\left(\frac{dy}{dx}\right)^2 = 7$

1st order
2nd degree

solution to a DE ≡ a relation between variables that satisfies the DE

note: doesn't have to be a function

general solution - a solution to a DE that contains a number of arbitrary constants equal to the order of that DE

2nd order DE → 2 arbitrary constants

particular solution - when specific values are given to at least one of the constants

example:

show that $y = c \ln x$ is a solution to the following DE: $y' \ln x - \frac{y}{x} = 0$

↑ some constant

$y = c \ln x$

$y' = \frac{c}{x}$

$$y' \ln x - \frac{y}{x} = 0$$

$$\frac{c}{x} \ln x - \frac{c \ln x}{x} = 0$$ ✓

note: this allows you to check your work!

by the way, was this solution a general solution or a particular solution?

general (1 constant, 1st order)

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example: show that $y = 3e^{2x}$ and $y = e^{2x} - 5$ are both solutions to the DE

$$y'' = 2y'$$

$y = 3e^{2x}$
$y' = 6e^{2x}$
$y'' = 12e^{2x}$

$y'' = 2y'$
$12e^{2x} = 2 \cdot 6e^{2x}$ ✓

$y = e^{2x} - 5$
$y' = 2e^{2x}$
$y'' = 4e^{2x}$

$y'' = 2y'$

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$y'' = 2y'$

$4e^{2x} = 2 \cdot 2e^{2x}$ ✓

what do you think the general solution might be, given these particular solutions?

$y = C_1 e^{2x} + C_2$