Section 31.3: Integrating Combinations

Friday, April 05, 2013
1:30 PM

Assisnment #1 due on
Tuesday, April 23rd

Quiz #1 on
Friday, April 26th

- covering sections 31.1 to 31.8 inclusive
- formula sheet has been posted

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review of differentials: dx, dy, dz

if $y = \sin x$, what is dy?

$dy = \cos x \, dx$

note: $dy = \frac{dy}{dx} dx$

if $y = x \sin x$, what is dy?

$dy = (x \cos x + \sin x) dx$

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if $z = x + y$. what is dz?

if $z = x + y$, what is dz?

$dz = dx + dy$

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if $z = xy$, what is dz?

$dz = y\,dx + x\,dy$

$z = x^2 + y^2$

$dz = 2x\,dx + 2y\,dy$

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note: a different way of writing the same thing is

$d(x^2+y^2) = 2x\,dx + 2y\,dy$

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examples of integrating combinations:

solve: $(2y + t)\,dy + y\,dt = 0$

$2y\,dy + t\,dy + y\,dt = 0$

method #1:

let $v = ty$

$dv = y\,dt + t\,dy$

so $2y\,dy + dv = 0$

$\int 2y\,dy + \int dv = \int 0$

$$y^2 + u = C$$

$$\boxed{y^2 + ty = C}$$

method #2:

$$2y\,dy + \boxed{t\,dy + y\,dt} = 0$$

$$2y\,dy + d(ty) = 0$$

$$y^2 + ty = C$$

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solve: $x\,dy - y\,dx + y^2\,dx = 0$

method #1

$$\frac{x\,dy - y\,dx}{y^2} + dx = 0$$

$$\int -d\left(\frac{x}{y}\right) + \int dx = \int 0$$

$$-\frac{x}{y} + x = C$$

scrap:

$$d\left(\frac{x}{y}\right) = \frac{y\,dx - x\,dy}{y^2}$$

method #2:

$$x\,dy - y\,dx + y^2\,dx = 0$$

$$\frac{x\,dy - y\,dx}{y^2} + dx = 0$$

let $u = \frac{x}{y}$

$$du = \frac{y\,dx - x\,dy}{y^2}$$

$$\int -du + \int dx = \int 0$$

$$\int \quad \int \quad \int$$

$$-u + x = C$$

$$-\frac{x}{y} + x = C$$

$$\sec(xy)\,dx + x\,dy + y\,dx = 0$$

let $u = xy$

$du = x\,dy + y\,dx$

$$\sec u\,dx + du = 0$$

$$dx + \frac{du}{\sec u} = 0$$

$$\int dx + \int \cos u\,du = \int 0$$

$$x + \sin u = C$$

$$x + \sin xy = C$$

$$x\,dy + y\,dx + 4xy^3\,dy = 0$$

let $u = xy$

$du = x\,dy + y\,dx$

$$du + 4xy^3\,dy = 0$$

$$du + (xy)\,4y^2\,dy = 0$$

$$du + u\,4y^2\,dy = 0$$

$$\frac{du}{u} + 4y^2\,dy = 0$$

$$\ln v + \frac{4}{3} y^3 = C$$

$$\ln xy + \frac{4}{3} y^3 = C$$