Section 31.5: Numerical Solutions of First-Order DEs

Tuesday, April 09, 2013
11:30 AM

note: in 8th edition, this is Supplement 8

suppose you have some differential equation

$$\frac{dy}{dx} = f(x, y)$$

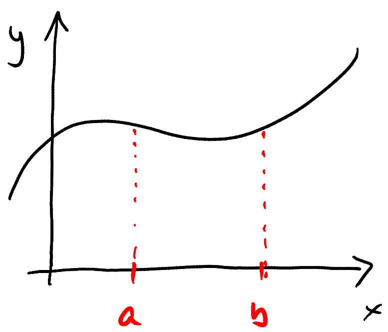
and you also know that when $\underbrace{x = x_0, \text{ that } y = y_0}_{\text{some initial condition}}$.

we'd like to solve for y and plug in our initial conditions to calculate values for our constants

→ but what if our usual methods (separation of variables, etc) fail?

numerical methods

consider some curve:



the fundamental theorem of calculus says that

$$y(x=b) - y(x=a) = \int_a^b \frac{dy}{dx}\, dx$$

so $$y(b) = y(a) + \int_a^b \frac{dy}{dx}\, dx$$

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but recall that our DE is

$$\frac{dy}{dx} = f(x,y)$$

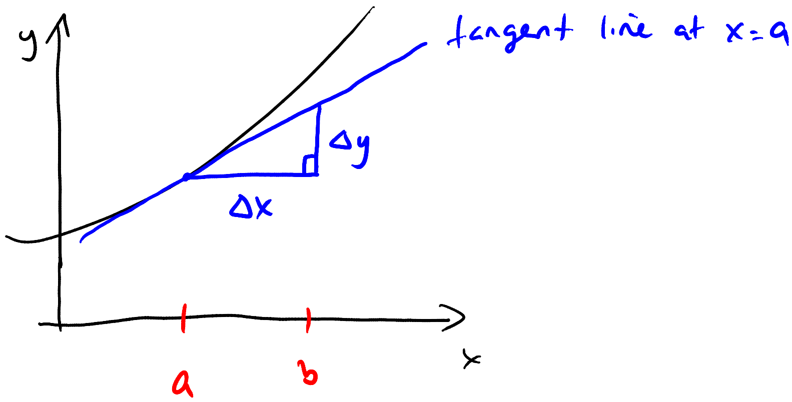
so $$y(b) = y(a) + \underbrace{\int_a^b f(x,y)\, dx}$$

if we could integrate
this directly, we'd be set
but if we can't ?

numerical methods use different approximations to
find this integral

you've actually studied one method already
→ linearization (Math 185)

linearization (Section 24.8)



$$y(b) = y(a) + \int_a^b \underbrace{\frac{dy}{dx}}\, dx$$

assume slope is constant
so $m = y'(a)$

$$y(b) = y(a) + m(b-a)$$

usually written

$$L(x) = y(a) + y'(a)(x-a)$$

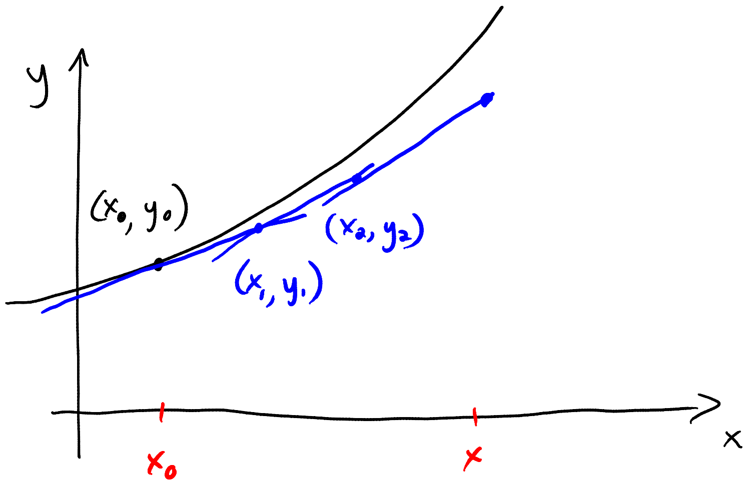
works well provided that

a) x is close to a

b) the curve is slowly varying

Euler's method (assignment question)

- very similar to linearization
- the difference is that you do a <u>series</u> of linearizations



so, what's y at this x?

then break down the integral into little steps of size Δx

$$y_1 = y_0 + \underbrace{y'(x_0)}_{\text{from DE}} \underbrace{(x_1 - x_0)}_{\Delta x}$$

$$y_2 = y_1 + y'(x_1)(x_2 - x_1)$$
$$= y_1 + f(x_1, y_1)\,\Delta x$$

$$y_3 = y_2 + f(x_2, y_2)\,\Delta x$$

Euler's method

$$\boxed{y_{n+1} = y_n + \Delta x\, f(x_n, y_n)}$$

error in your estimate is on the order of Δx

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full example:

Use Euler's method to find the y-values for the DE

$$\frac{dy}{dx} = \ln(x+y)$$

given that the curve of the solution passes through the point $(0,1)$. Use a step size of 0.1 and calculate values of y for $x = 0$ to $x = 0.5$

answer: set up a table

$\Delta x = 0.1$

$f(x,y) = \ln(x+y)$

new y = old y + $f(x,y)\,\Delta x$

x	y	$f(x,y)$	$f(x,y)\cdot\Delta x$	new y

0	1	0	0	1
0.1	1	0.09531	0.009531	1.009531
0.2	1.009531	0.190233	0.0190233	1.02855
0.3	1.02855	0.284091	0.028409	1.05696
0.4	1.05696	0.376351	0.0376351	1.0946
0.5	1.0946			

these points are my answer