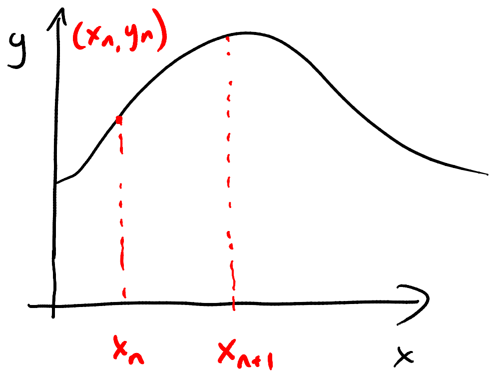
Section 31.5: cont'd:

Wednesday, April 10, 2013
11:34 AM

Euler's method: easy to visualize, but not so accurate

Improved Euler: (aka Heun's method)

DE: $\frac{dy}{dx} = f(x, y)$



Euler uses the slope at x_n to calculate where y_{n+1} will be

$$y_{new} = y_{old} + f(x, y)\Delta x$$

Improved Euler takes the average of the slopes at each end instead (well, sort of)

→ problem: we don't actually know what y_{n+1} is, but we can estimate it using Euler, then calculate the slope at our estimated point

note: this type of method is called a "predictor-corrector" method (guess-and-adjust) where Euler is "predictor" only

$$x_{n+1} = x_n + \Delta x$$

$$k_1 = f(x_n, y_n) \leftarrow \text{slope at } x_n$$

$$\underbrace{k_2}_{\substack{\text{estimate}\\ \text{of slope}\\ \text{at } x_{n+1}}} = f\Big(x_{n+1},\ \underbrace{y_n + f(x_n, y_n)\,\Delta x}_{\text{estimate of } y_{n+1}}\Big)$$

$$y_{n+1} = y_n + \underbrace{\frac{k_1 + k_2}{2}}_{\substack{\text{average of the}\\ \text{two slopes}}} \Delta x$$

this method is much better than Euler, but still not fabulous

__Runge - Kutta method:__

$$x_{n+1} = x_n + \Delta x$$

$$y_{n+1} = y_n + \underbrace{\frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)}_{\substack{\text{improved Euler}\\ \text{on steroids}}} \Delta x$$

where $k_1 = f(x_n, y_n)$ ← slope on LHS

$$k_2 = f\left(x_n + \frac{\Delta x}{2},\ y_n + \frac{k_1 \Delta x}{2}\right)$$

first guess at slope halfway between our points

$$k_3 = f\left(x_n + \frac{\Delta x}{2},\ y_n + \frac{k_2 \Delta x}{2}\right)$$

another guess at slope halfway

better guess at slope halfway between our points

$$k_4 = f(x_n + \Delta x, \; y_n + k_3 \Delta x)$$

guess at the slope on the RHS

this method is commonly called RK4