

Section 31.6: cont'd:

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1:30 PM

Most of the time, when we study objects in freefall, we neglect air resistance. Why?

- a) for certain conditions, air resistance is very small and can be ignored
and compact
- small objects falling through short distances

- b) $F_{\text{drag}} \propto v$ so mathematically difficult to handle if you don't have DEs

note: $F \propto v$ only at low speeds
 $\propto v^2$ at higher speeds

example: Find a relationship for $v(t)$ for an object starting at rest and falling under the influence of gravity, given that the air resistance is proportional to the speed of the object through the air.



$$F_{\text{drag}} \propto v$$

$$F_{\text{drag}} = \underset{\substack{\uparrow \\ \text{same constant}}}{k} v$$

$$\sum \vec{F} = m \vec{a}$$

$$mg - kv = ma$$

$$mg - kv = m \frac{dv}{dt} \quad \} \text{ DE}$$

$$\frac{dv}{dt} = g - \frac{k}{m} v$$

$$\int \frac{dv}{g - \frac{k}{m} v} = \int dt$$

note: g , k , and m are constants

$$-\frac{m}{k} \ln \left(g - \frac{k}{m} v \right) = t + C$$

$$\ln \left(g - \frac{k}{m} v \right) = -\frac{k}{m} t - \frac{k}{m} C$$

$$g - \frac{k}{m} v = e^{-\frac{k}{m} t - \frac{kC}{m}}$$

$$g - \frac{k}{m} v = e^{-\frac{k}{m} t} e^{-\frac{kC}{m}} \quad \text{another constant}$$

$$g - \frac{k}{m} v = C_1 e^{-\frac{k}{m} t}$$

$$-\frac{k}{m} v = C_1 e^{-\frac{k}{m} t} - g$$

$$v = -\frac{mC_1}{k} e^{-\frac{k}{m} t} + \frac{gm}{k}$$

at $t=0$, $v=0$

$$0 = -\frac{mC_1}{k} e^0 + \frac{gm}{k}$$

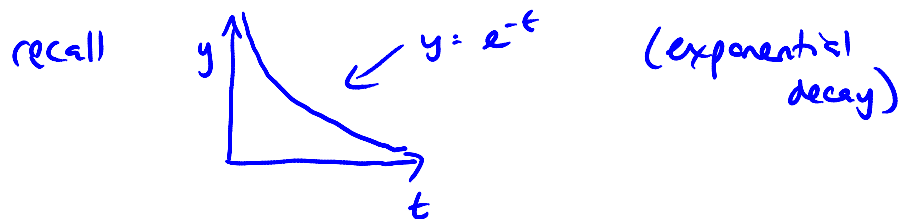
$$\frac{mC_1}{k} = \frac{gm}{k}$$

$$c_1 = g$$

So

$$\begin{aligned} v &= \frac{gm}{k} - \frac{gm}{k} e^{-\frac{k}{m}t} \\ &= \frac{mg}{k} \left(1 - e^{-\frac{k}{m}t} \right) \end{aligned}$$

what happens at $t \rightarrow \infty$?



$$\text{as } t \rightarrow \infty, e^{-t} \rightarrow 0$$

$$\lim_{t \rightarrow \infty} v = \lim_{t \rightarrow \infty} \frac{mg}{k} \left(1 - e^{-\frac{k}{m}t} \right)$$

$$v \rightarrow \frac{mg}{k}$$

terminal velocity

exponential decay:

The rate of decay of radioactive atoms is proportional to the amount of radioactive material. Find an expression for the amount of material at time t , $N(t)$, given that the initial amount of material is N_0 .

the "rate of change" means $\frac{d}{dt}$

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -kN \quad \leftarrow \text{separable!}$$

$$\int \frac{dN}{N} = \int -k dt$$

$$\ln N = -kt + C$$

$$N = e^{-kt+C}$$

$$= e^{-kt} e^C = C_1$$

$$N = C_1 e^{-kt}$$

at $t=0$, $N = N_0$

$$N_0 = C_1 e^{\cancel{0}}$$

$$N = N_0 e^{-kt}$$