Section 31.7 & 31.8: cont'd

Tuesday, April 16, 2013
11:32 AM

recall from yesterday:

$$a_1 y'' + a_2 y' + a_3 y = b$$

→ 2nd order linear DE

→ if $b = 0$, then it's also homogeneous

---

follow along on handout to solve:

$$y'' - 2y' - 8y = 0$$

step ①: write the auxiliary equation:

$$m^2 - 2m - 8 = 0$$

and solve for m: (either by factoring or quadratic formula)

$$(m-4)(m+2) = 0$$

$$m = -2, 4$$

note: two real roots for this situation

step ②: write the solution:

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$= C_1 e^{4x} + C_2 e^{-2x}$$

$$= C_1 e^{4x} + C_2 e^{-2x}$$

this is the general solution to the given DE

---

let's check our answer!

$$y = C_1 e^{4x} + C_2 e^{-2x}$$

$$y' = 4C_1 e^{4x} - 2C_2 e^{-2x}$$

$$y'' = 16C_1 e^{4x} + 4C_2 e^{-2x}$$

original DE:

$$y'' - 2y' - 8y = 0$$

$$(16C_1 e^{4x} + 4C_2 e^{-2x}) - 2(4C_1 e^{4x} - 2C_2 e^{-2x})$$

$$-8(C_1 e^{4x} + C_2 e^{-2x}) = 0$$

$$0 = 0 \quad \checkmark$$

---

solve the following DE:

$$y'' - 2y' - 6y = 0$$

auxiliary equations:

$$m^2 - 2m - 6 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 + 24}}{}$$

$$= \frac{2 \pm \sqrt{28}}{2}$$

$$= \frac{2 \pm 2\sqrt{7}}{2}$$

$$= 1 \pm \sqrt{7}$$

$\therefore$

$$y = C_1 e^{(1+\sqrt{7})x} + C_2 e^{(1-\sqrt{7})x}$$

---

note: if I asked you to solve

$y''' - 2y'' - 6y' = 0$ instead, then

how would your solution change?

auxiliary equation: $m^3 - 2m^2 - 6m = 0$

$$m(m^2 - 2m - 6) = 0$$

$$m = 0,\ 1 \pm \sqrt{7}$$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x}$$

$$= C_1 e^{0x} + C_2 e^{(1+\sqrt{7})x} + C_3 e^{(1-\sqrt{7})x}$$

$$= C_1 + C_2 e^{(1+\sqrt{7})x} + C_3 e^{(1-\sqrt{7})x}$$

Solve the following DE, given that when $x = 0$, $y = 1$ and $y' = 2$.

$$y'' + y' = 0$$

auxiliary equation: $m^2 + m = 0$

$$m(m+1) = 0$$

$$m = 0, -1$$

general solution: $y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$$= c_1 e^0 + c_2 e^{-x}$$

$$= c_1 + c_2 e^{-x}$$

particular solution:

$$y = c_1 + c_2 e^{-x}$$

when $x = 0$, $y = 1$:

$$1 = c_1 + c_2$$

$$y' = -c_2 e^{-x}$$

when $x = 0$, $y' = 2$

$$2 = -c_2$$

$$c_2 = -2$$

so plug back in

$$c_1 = 3$$

so the particular solution is

$$\boxed{y = 3 - 2e^{-x}}$$

let's check our answer:

$$y = 3 - 2e^{-x}$$

$$y' = 2e^{-x}$$

$$y'' = -2e^{-x}$$

$$y'' + y' = 0$$

$$-2e^{-x} + 2e^{-x} = 0 \quad ✓$$

---

solve the following DE:

$$y'' - 6y' + 9y = 0$$

auxiliary equation:

$$m^2 - 6m + 9 = 0$$

$$m = 3$$

so <u>one</u> real solution to the aux equation

so proceed to step 3 on the handout:

$$y = (c_1 + c_2 x)e^{mx}$$

$$\boxed{y = (c_1 + c_2 x)e^{3x}}$$

note: why won't the previous solution work?

note: why won't the previous solution work?

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$
$$= c_1 e^{3x} + c_2 e^{3x}$$
$$= (c_1 + c_2) e^{3x}$$
$$= c_3 e^{3x}$$

too few constants! need 2 for 2nd order

let's check:

$$y = (c_1 + c_2 x) e^{3x}$$
$$y' = (c_1 + c_2 x) 3e^{3x} + c_2 e^{3x}$$
$$y'' = (c_1 + c_2 x) 9e^{3x} + \underbrace{c_2 3e^{3x} + 3c_2 e^{3x}}_{= 6c_2 e^{3x}}$$

$$y'' - 6y' + y = 0$$

$$9(c_1 + c_2 x) e^{3x} + 6c_2 e^{3x} - 6\left[(c_1 + c_2 x) 3e^{3x} + c_2 e^{3x}\right]$$
$$+ 9(c_1 + c_2 x) e^{3x} = 0$$

$\checkmark$