Section 31.7 & 31.8: cont'd

Wednesday, April 17, 2013
11:29 AM

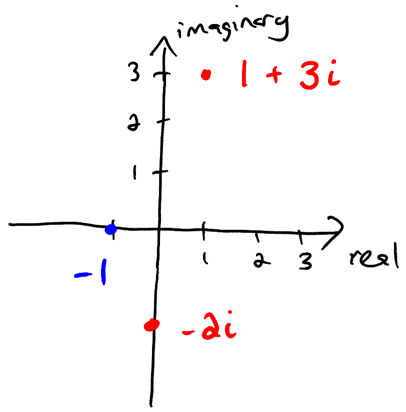
complex numbers:

$i = \sqrt{-1}$ (Washington uses j)

complex numbers - can be written as

$a + bi$

where a and b are real numbers



complex plane

addition and subtraction:

$(1 + 3i) + (2 - 5i) = 3 - 2i$

$(1 + 3i) - (2 - 5i) = -1 + 8i$

simplification:

$\sqrt{-4} = \sqrt{4}\sqrt{-1}$

$= 2i$

which means that

$$x^2 = -9$$

$$x = \pm\sqrt{-9}$$

$$= \pm\sqrt{9}\sqrt{-1}$$

$$= \pm 3i$$

by the way,

$$i = \sqrt{-1}$$

and $i^2 = -1$

which means that $i^3 = i^2 \cdot i$

$$= -1 \cdot i$$

$$= -i$$

$$i^4 = i^2 \cdot i^2$$

$$= (-1)(-1)$$

$$= 1$$

so, solve:

$$x^2 + 4x + 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$= -2 \pm i$

Why do we care in Math 189?

- Can get complex answers when you solve the auxiliary equation

<u>but</u> what does it look like when you raise e to a complex power?

Maclaurin series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{ix} = 1 + (ix) + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \cdots$$

$$= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)$$

$$= \cos x + i \sin x$$

so $$e^{(2+i)x} = e^{2x} e^{ix}$$

$$= e^{2x}(\cos x + i \sin x)$$

so, using all of this, let's do an example of a DE:

$$y'' + 25y = 0$$

auxiliary equation: $m^2 + 25 = 0$

$$m^2 = -25$$

$$m = \pm\sqrt{-25}$$

$$= \pm 5i$$

so go to step 4 of handout:

$m = \alpha \pm \beta i$ where $\alpha = 0$, $\beta = 5$

plug in: $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

$$= e^0 (c_1 \cos 5x + c_2 \sin 5x)$$

$$\boxed{y = c_1 \cos 5x + c_2 \sin 5x}$$

check:

$$y = c_1 \cos 5x + c_2 \sin 5x$$

$$y' = -5c_1 \sin 5x + 5c_2 \cos 5x$$

$$y'' = -25c_1 \cos 5x - 25c_2 \sin 5x$$

$$y'' + 25y = 0$$

$$-25c_1 \cos 5x - 25c_2 \sin 5x + 25(c_1 \cos 5x + c_2 \sin 5x) = 0 \checkmark$$

Solve the following DE:

$$y'' - 6y' + 34y = 0$$

auxiliary equation: $m^2 - 6m + 34 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{-100}}{2}$$

$$= \frac{6 \pm 10i}{2}$$

$$= 3 \pm 5i$$

So complex solutions $m = \alpha \pm \beta i$

where $\alpha = 3$ and $\beta = 5$

and $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

$$y = e^{3x}(c_1 \cos 5x + c_2 \sin 5x)$$

Solve the DE:

$$25y'' + 40y' + 16y = 0$$

auxiliary equation: $25m^2 + 40m + 16 = 0$

$$(5m + 4)^2 = 0$$

$$m = -\frac{4}{5}$$

one repeated real m

$$\boxed{y = (c_1 + c_2 x) e^{-4/5 x}}$$