Section 31.9: Solutions of Nonhomogeneous Linear 2nd order DEs

Friday, April 19, 2013
2:02 PM

nonhomogeneous:

$$ay'' + by' + cy = \underbrace{f(x)}_{\text{for homogeneous this was zero}}$$

here's the plan:

we find y_c ← called the complementary solution - it's the general solution to the associated homogeneous DE

→ set RHS to zero and solve, this solution is y_c

then $y = y_c + y_p$

y: the solution to the DE

y_c: solution to the homogeneous

y_p: particular solution necessary to get the RHS of the nonhomogeneous solution

note: y_p will have no arbitrary constants so, how do we find y_p?

y_p is an expression that contains all possible forms of f(x) and their derivatives

if $f(x) = x^2$, then $y_p = C_1 x^2 + C_2 x + C_3$

$f(x) = x^3$, then $y_p = C_1 x^3 + C_2 x^2 + C_3 x + C_4$

$f(x) = e^x$, then $y_p = C_1 e^x$

↑ all derivatives are of this form

$f(x) = x e^x$, then $y_p = C_1 x e^x + C_2 e^x$

$f(x) = \sin x$, then $y_p = C_1 \sin x + C_2 \cos x$

so, if y_p is then a particular solution, how do you find the constants?

→ this method is called "the method of undetermined coefficients"

→ substitute in DE