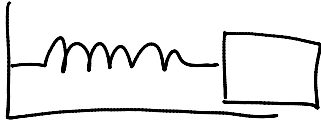


Section 31.10: cont'd

Wednesday, April 24, 2013
12:03 PM

Hooke's Law: case of no friction, no external force (once mass was let go), ideal spring



$$-kx = m \frac{d^2x}{dt^2}$$

Rearranging:

$$m \frac{d^2x}{dt^2} + kx = 0 \quad \text{homogeneous}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m} x = 0$$

use n instead of m for aux eqn
(because $m = \text{mass}$)

$$n^2 + \frac{k}{m} = 0$$

$$n^2 = -\frac{k}{m}$$

$$n = \pm \sqrt{\frac{k}{m}} i$$

so $n = \alpha \pm \beta i$ where $\alpha = 0$ and $\beta = \sqrt{\frac{k}{m}}$

$$x = e^{\alpha t} (C_1 \cos \beta t + C_2 \sin \beta t)$$

$$= C_1 \cos\left(\sqrt{\frac{k}{m}} t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}} t\right)$$

note: this is the same result used in Phys 192
where

$$\omega_0 = \sqrt{\frac{k}{m}}$$

and you write

$$x = C_1 \cos \omega_0 t + C_2 \sin \omega_0 t$$

ω_0 is called the natural frequency
of oscillation

\Rightarrow Simple harmonic motion