Section 31.10: cont'd

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11:36 AM

A marble rolling back and forth in the bottom of a circular bowl follows a path given by:

$$\frac{d^2s}{dt^2} + 4s = 0$$

where s is the arclength in cm, and t is the time in seconds.

a) Find the general solution to the DE.

b) If the marble is displaced by 2.5 cm and then let go, what is the arclength s as a function of time?

c) What if, instead, the marble is started at the bottom of the bowl but is given a push such that its initial speed is 6 cm/s? What is $s(t)$ then?

a) aux eqn: $m^2 + 4 = 0$

$m = \pm 2i$ $\quad \alpha = 0$, $\beta = 2$

$$s = e^{\alpha t}(C_1 \cos \beta t + C_2 \sin \beta t)$$

$$= C_1 \cos 2t + C_2 \sin 2t$$

b) at $t = 0$, $s = 2.5$ cm and $\frac{ds}{dt} = 0$

$$\frac{ds}{dt} = -2C_1 \sin 2t + 2C_2 \cos 2t$$

or

$0 = -2C_1 \sin 0^{0} + 2C_2 \cos 0^{1}$

so $C_2 = 0$

$s = C_1 \cos 2t + C_2^{0} \sin 2t$

$2.5 = C_1 \cos 0^{1} + C_2 \sin 0^{0}$

$C_1 = 2.5$

$s = 2.5 \cos 2t$

c) at $t = 0$, $s = 0$ and $\frac{ds}{dt} = 6$ cm/s

$s = C_1 \cos 2t + C_2 \sin 2t$

$0 = C_1 \cos 2t^{1} + 0$

$s = C_2 \sin 2t$

$\frac{ds}{dt} = 2C_2 \cos 2t$

$6 = 2C_2$

$C_2 = 3$

$s = 3 \sin 2t$