Section 3.3: Tchebysheff and the Empirical Rule

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2:34 PM

Tchebysheff's theorem: works for all distributions (symmetrical or skewed, unimodal or multimodal)

- for any set of measurements,

at least $\left(1 - \frac{1}{k^2}\right)$ of the measurements

will lie within k standard deviations of the mean for $k \geq 1$.

k	$1 - 1/k^2$
1	0
1.5	5/9
2	3/4
2.5	21/25
3	8/9

totally useless statement

so $\geq 0\%$ lie within $\mu \pm 1\sigma$

$\geq 55.\bar{5}\%$ $\mu \pm 1.5\sigma$

$\geq 75\%$ $\mu \pm 2\sigma$

$\geq 84\%$ $\mu \pm 2.5\sigma$

$\geq 88.\bar{8}\%$ $\mu \pm 3\sigma$

population

or sample

$\bar{x} \pm 3s$

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the Empirical Rule:

- only works for "mound-shaped" data sets

mound-shaped: unimodal and roughly symmetrical

approximately 68% of measurements lie within $\mu \pm 1\sigma$

95% $\mu \pm 2\sigma$

99.7% $\mu \pm 3\sigma$

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look at handout:

Fictitious Bimodal Distribution

k	$\bar{x}-ks$	$\bar{x}+ks$	Actual number	Actual %	Tcheby	Empirical
1	1.57	7.93	10	50 %	≥0 % ✓	~68 % ✗
2	-1.60	11.10	20	100 %	≥75% ✓	~95% sort of
3	-4.78	14.28	20	100 %	≥ 90 %	~99.7% ✓

note: should Tcheby work? yes (always true)

should Empirical work? no (not unimodal)

k	$\bar{x}-ks$	$\bar{x}+ks$	actual number	actual %	Tcheby	Empirical
1	4.46	8.54	28	66.7%	≥ 0 % ✓	~68 % ✓
2	2.42	10.58	40	95.2%	≥ 75 % ✓	~95 % ✓
3	0.38	12.62	42	100.0%	≥ 90 % ✓	~99.7% ✓

the distribution is mound-shaped so both Tchebysheff and Empirical should work (and do)