Section 4.1 : Cont'd:

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11:34 AM

mutually exclusive - two events are mutually exclusive if when one event occurs, the other cannot occur

example: rolling 6-sided die

are the following pairs of events mutually exclusive?

{ rolling an odd number
  rolling a 2 ✓

note: mutually exclusive events don't have to span the sample space - there can be other events left over

{ rolling an odd number
  " " even ✓

{ rolling a 1 or a 2
  rolling ≥ 2 ✗

note: simple events are always mutually exclusive!

simple events: 1, 2, 3, 4, 5, 6

↑ if you've rolled a 2, you haven't rolled any other value

sample space ≡ the set of all simple events

(the complete set of experimental outcomes)

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example: rolling a pair of 4-sided dice
what is the sample space?

sample space:

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

note: the sample space is a list of all possible events

a total of 16 simple events

note: if the two dice are fair (equal chance of landing on any of the sides), then the probability of any single event is 1/16

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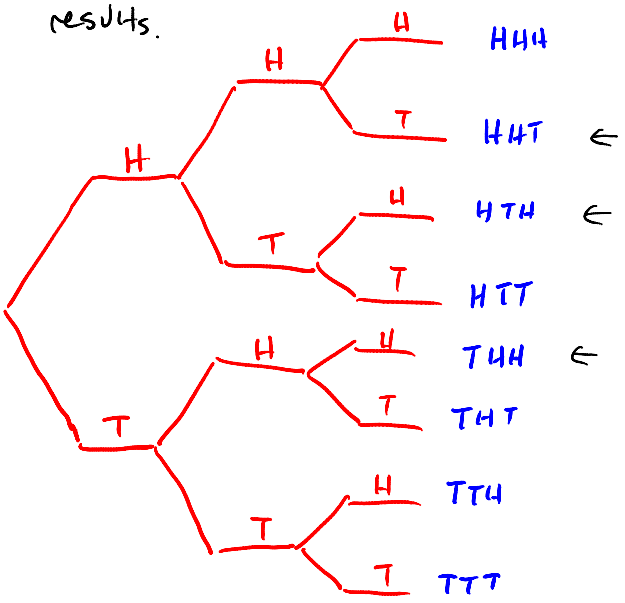
if you are having trouble generating the sample space, can always consider a tree diagram:

example: write out the sample space for flipping a coin 3 times and recording the results.



note: how many ways of getting only 1 tail? 3

if the coin is fair, then the odds of getting one tail are $\frac{3}{8}$

Classical probability: probability of an event happening equals the sum of the probabilities of all of the simple events in that event

$P(A)$ - probability of event A happening

from our previous example:

$$P(\text{only one tail}) = P(HHT) + P(HTH) + P(THH)$$
$$= 1/8 + 1/8 + 1/8$$
$$= 3/8$$

if all simple events are equally likely, then

$$P(A) = \frac{n(A)}{n}$$

where $n(A)$ = number of ways event A can happen

n = total number of events

two properties of probability:

$$0 \leq P(A) \leq 1$$

0: never happens

1: always happens

$$\sum_i P(A_i) = 1$$

the sum of all probabilities for a sample space

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example: Roll a fair four-sided die twice. What's the probability that the sum of the rolls is six or more?

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

$P(\text{sum} \geq 6) = \frac{6}{16} = \frac{3}{8}$

complement:

the complement of event A can be written as

$\bar{A}$ ← I will use "not A"

other notations:

$\sim A$

A^c

$\neg A$

what is a complement? The complement of A is the set of simple events in which A does not occur

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

Example from laziness:

If you roll a fair 4-sided die twice, what is the probability that the sum is 5 or lower?

$$P(\text{sum} \leq 5) = 1 - \underbrace{P(\text{sum} \geq 6)}_{\text{already calculated!}}$$

$$= 1 - 3/8$$

$$= 5/8$$

If one fair 4-sided die is rolled twice, what's the probability of rolling two different numbers?

$$P(\text{different}) = 1 - P(\text{same})$$

$$= 1 - 4/16$$

$$= 3/4 = 75\%$$