Section 4.2: Useful Counting Rules

Friday, May 10, 2013
1:38 PM

The addition rule:

How many numbers from 1 to 20 inclusive are

a) divisible by 2 ?
b) " " 3 ?
c) " " 2 or 3?

a) divisible by 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

so 10 numbers

b) divisible by 3: 3, 6, 9, 12, 15, 18

so 6 numbers

c) divisible by 2 or 3: 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

so (13) numbers

why not 16? because there's overlap in the two lists

note: 3 entries overlap

so $n(A \text{ or } B) = n(A) + n(B) - n(AB)$

$n(AB)$ = $n(A \text{ and } B)$

and $P(A \text{ or } B) = P(A) + P(B) - P(AB)$

if $P(AB) = 0$, then events A and B are mutually exclusive

digression: set notation (I will not use this)

$A \cup B$ = A union B = A or B

$A \cap B$ = A intersect B = A and B = AB

multiplication rule:

consider an experiment that is performed in k steps and all of the steps are independent (result of step 1 does not influence the outcome of step 2)

then the total number of possible outcomes for the experiment is:

$$n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k$$

(n_k: the number of outcomes for step k)

example: what is the total number of BC licence plates for cars (ignoring reserved words and personalized plates)?

patterns: number - number - number - letter - letter - letter
letter - letter - letter - number - number - number

multiplication rule:

$$\underline{10}\ \underline{10}\ \underline{10}\ \underline{26}\ \underline{26}\ \underline{26} = 10^3 \cdot 26^3$$

$$= 17\,576\,000$$

so total number of plates $= 2(17\,576\,000)$

$$= 35\,152\,000$$

---

example: In the mythical Canadian province of Gondor, licence plates are issued with the pattern letter-letter-letter-number-number. Due to recent political events, the combination EYE is no longer allowed. How many legal licence plates are there?

_ _ _ _ _

total number of licence plates: $26^3 \cdot 10^2$

number of "illegal" plates: $1 \cdot 10^2$

number of "legal" plates: $26^3 \cdot 10^2 - 10^2$

$= 1757\,500$

---

example:

How many 5-digit case-sensitive alphanumeric passwords are there that:

a) start or end with an A

b) contain <u>at least</u> one letter <u>and</u> one number

how many allowed characters? 26 (lower case) $+ 26$ (upper case) $+ 10$ (digits)

$= 62$

a) $n(\text{start or end with A}) = n(\text{start with A}) + n(\text{end with A}) - n(\text{start and end with A})$

$$= 62^4 + 62^4 - 62^3$$

$$= 29\ 314\ 344$$

start with A: A ____ 62^4

end with A: 62^4

start and end with A: A___ A 62^3

b) total legal = total − illegal

$$= 62^5 - (52^5 + 10^5)$$

all letters, all numbers

$$= 535{,}828{,}800$$