Section 4.3: Conditional Probability and Independence

Monday, May 13, 2013
1:54 PM

conditional probability

$P(B \mid A)$ – the probability of B happening if A has already happened

↑ "if"

example:

let F = flossing regularly
C = getting cavities

then $P(C \mid F)$ is the probability of getting cavities if you floss regularly

– in this case, it's likely that

$P(C \mid F) < P(C)$, no?

note: correlation does not imply causation!
just because two things are linked does not mean that one causes another

so, how do you calculate conditional probabilities?

$$P(C \mid F) = \frac{P(CF)}{P(F)} = \frac{n(CF)}{n(F)}$$

which leads to another rule:

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$$P(AB) = P(B|A)\,P(A)$$

Contingency tables handout

① $P(T) = \dfrac{n(T)}{n} = \dfrac{50}{100} = 50\%$

② $P(FB) = \dfrac{n(FB)}{n} = \dfrac{25}{100} = 25\%$

③ $P(M \text{ or } B) = \dfrac{n(MB) + n(MT) + n(FB)}{n}$

$= \dfrac{25 + 45 + 25}{100}$

$= 95\%$

④ $P(T \mid F) = \dfrac{n(FT)}{n(F)} = \dfrac{5}{30} = \dfrac{1}{6}$

⑤ $P(F \mid T) = \dfrac{n(FT)}{n(T)} = \dfrac{5}{50} = \dfrac{1}{10}$

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independent vs. dependent events:

- consider two events A & B.

- if A is just as likely when you look at the entire population as when you look only at subpopulation B, then we say the events are independent

example: let C: getting cavities
F: flossing

if the probability of getting cavities is the same when you look at the entire population as when you look at only the people who floss regularly, then these events are independent

another nice way of asking the same question:

Does the probability of getting cavities <u>depend</u> on whether you are looking at just the flossers or everybody?

If YES, events are dependent.
If NO, events are independent.

so, how do you determine independence? you compare probabilities.

for our cavities/flossing example:

if $P(C) = P(C \mid F)$, then independent

or

if $P(F) = P(F \mid C)$, then independent

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back to the handout

last question:

if $P(F \mid T) = P(F)$, then independent

$$\left.\begin{aligned} P(F \mid T) &= 10\% \\ P(F) &= 30\% \end{aligned}\right\} \text{not the same}$$

conclusion: dependent!