Section 4.4: Bayes' Theorem

Monday, May 13, 2013
2:48 PM

Suppose you have a situation in which you are given $P(A)$ and $P(B|A)$, but you want to calculate $P(A|B)$ instead. How can you do that easily?

- there's a single calculation you can do (detailed at end of lecture), or you can take a step-by-step approach

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step-by-step approach:

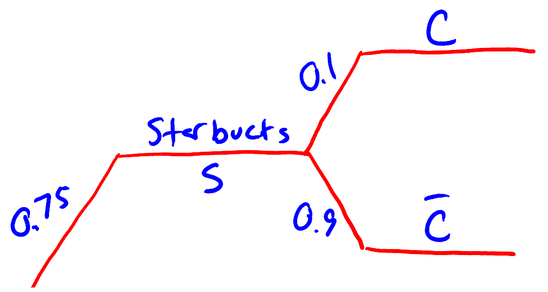
example: Suppose there are only two coffee houses in Cook St. Village: Starbucks and Moka House. If Gilles goes to Starbucks, he'll order a cappuccino 10% of the time. At Moka House, he'll order that same drink 40% of the time. Gilles goes to Starbucks 75% of the time.

a) What's the probability that when Gilles visits one of these coffee houses, he orders a cappuccino? $\Leftarrow$ trying to find $P(C)$

b) If he's ordered a cappuccino, what's the probability that he's at Starbucks? $P(S|C)$

given:

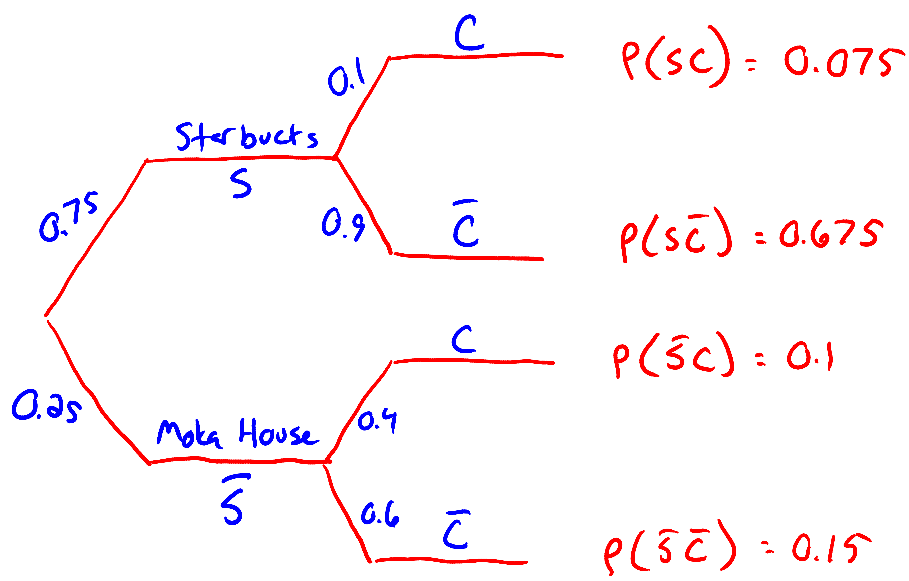
$$P(C|S) = 10\%$$
$$P(C|\bar{S}) = 40\%$$
$$P(S) = 75\%$$



$P(SC) = 0.075$

$P(S\bar{C}) = 0.675$

$P(S) = 75\%$



a) $P(C) = P(SC) + P(\bar{S}C)$

$= 0.075 + 0.1$

$= 0.175 = 17.5\%$

b) $P(S \mid C) = \dfrac{P(SC)}{P(C)} = \dfrac{0.075}{0.175} \approx 42.9\%$

by the way, are the events "ordering a cappuccino" and going to Starbucks independent ?

if independent, then: $P(C) = P(C \mid S)$

$P(C) = 17.5\%$
$P(C \mid S) = 10\%$ } different

$\therefore$ dependent

example: The test for a rare disease has 99% reliability. Only one percent of the population has this rare disease.

If the entire population is tested, then some who are healthy will have the test be positive for the disease. (false positive). Some who have the disease but will test negative (false negative).

→ If an individual test positive, what is the probability that they actually have this rare disease?

(hint: if you prefer, consider the population to be 10 000 individuals and determine how many individuals fall into each group)