

## Section 4.6: Combinations and Permutations

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11:52 AM

factorials:  $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

example: How many 4-digit PINs are there if  
a) repetition is allowed  
b) repetition is not allowed

$$a) \quad \frac{\quad}{10} \frac{\quad}{10} \frac{\quad}{10} \frac{\quad}{10} = 10^4$$

$$b) \quad \frac{\quad}{10} \frac{\quad}{9} \frac{\quad}{8} \frac{\quad}{7} \\ = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

$$\text{note: } 10 \cdot 9 \cdot 8 \cdot 7 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ = \frac{10!}{6!}$$

permutation: an ordered arrangement of  $r$  objects  
chosen from  $n$  possibilities without  
replacement

example: 4 digit PIN = picking 4 digits  
without repetition from 10 possibilities

notation:  $nPr$  ← I will use

$$P_r^n$$

$$P(n, r)$$

$${}^n P_r = \frac{n!}{(n-r)!}$$

example: How many 5-digit case-sensitive alphanumeric passwords are there if repetition is not allowed?

$$\begin{aligned} {}^n P_r &= {}_{62} P_5 \\ &= 776\,520\,240 \end{aligned}$$

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combinations - an unordered arrangement of  $r$  objects chosen without replacement from  $n$  possibilities

notation:  ${}^n C_r$  ← I will use

$$C_r^n$$

$$C(n, r)$$

$$\binom{n}{r}$$

← Schaum's Outline uses this

$${}^n C_r = \frac{n!}{\underbrace{r!}_{\text{circled}} (n-r)!}$$

↑  
 $r!$  is the number of ways

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you can arrange  $r$  objects

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example:

Pat is ordering a pizza from Penago. She has the option of three toppings chosen from a list of 15. How many different pizzas could she potentially order?

$${}_{15}C_3 = 455$$

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In the BC 6/49 lottery, the customer chooses without replacement six numbers from 49 possibilities. These numbers are then compared with the six numbers chosen at random at the end of the week.

How many different tickets could you choose that end up with 4 winning numbers?

$$\underbrace{{}_6C_4}_{\text{choosing 4 winning numbers from 6}} \cdot \underbrace{{}_{43}C_2}_{\text{choosing 2 losing numbers from 43}} = 15\,903 = 13\,545$$

how many tickets are there in total?

$${}_{49}C_6 = 13\,983\,816$$