Section 5.1: Discrete Random Variables

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11:39 AM

a variable x is a random variable if the value it assumes in the outcome of an experiment is a chance or random event

discrete random variable → quantitative

discrete → behaves like integers (as opposed to real numbers)

probability distribution for a discrete random variable is a formula, graph, or table that gives the possible values of x and their associated probabilities p(x)

note: the values of x must be mutually exclusive events

also: $0 \le p(x) \le 1$

$$\sum p(x) = 1$$

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example: Two 4-sided dice are rolled. Calculate the probability distribution for the sum of the two rolls.

brute force:

11 12 13 14
21 22 23 24

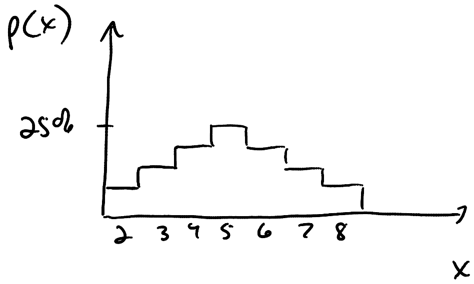
total number of events: 16

31 32 33 34
41 42 43 44

X = sum	p(x)	
2	1/16	
3	2/16	= 1/8
4	3/16	
5	4/16	= 1/4
6	3/16	
7	2/16	= 1/8
8	1/16	

perfectly acceptable answer

we can also graph this:



unimodal
symmetrical

what is the average value of the distribution?

population mean – also called the "expectation value" or "expected value"

mean
$$\mu = E(x) = \Sigma \, x \, p(x)$$

variance
$$\sigma^2 = E\left[(x-\mu)^2\right]$$

$$= \sum (x-\mu)^2 \, p(x)$$

formal definition
(annoying to calculate)

$$= \sum x^2 \, p(x) - \mu^2$$

calculation formula
(less annoying to calculate)

Standard deviation $\sigma = \sqrt{\sigma^2}$

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example: What is the expected value for winning the jackpot on BC Lotto 6/49 when the pot is 2.2 million dollars? Assume that there is only one winning ticket and it costs only \$2 to play?

how many tickets? $\quad {}_{49}C_6 = 13\ 983\ 816$

odds of winning jackpot? $\quad \dfrac{1}{13\ 983\ 816}$

x : either you lose your \$2

or you win \$2.2 million (though you have still paid \$2)

x	$p(x)$
-2	$\frac{13983815}{13983816}$
$2.2\times10^6 - 2$	$\frac{1}{13983816}$

$$E(x) = \sum x\, p(x)$$

$$= -2\left(\frac{13983815}{13983816}\right) + \left(2.2\times10^6 - 2\right)\frac{1}{13983816}$$

$$= -2 + \frac{2.2\times10^6}{13983816}$$

$$= -1.84$$

On average, including the jackpot winners, you lose \$1.84 every time you play.

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example: consider the probability distribution given below. Calculate the mean and standard deviation of x.

x	$p(x)$
0	$1/10$
1	$6/10$
2	$3/10$

$$E(x) = \sum x\, p(x)$$

$= 0 + 1 \cdot \frac{6}{10} + 2 \cdot \frac{3}{10}$

$= 1.2$

$\sigma^2 = \Sigma x^2 p(x) - \mu^2$

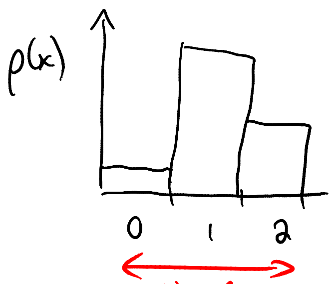
$= 0 + 1 \cdot \frac{6}{10} + 2^2 \cdot \frac{3}{10} - (1.2)^2$

$= 0.36$

$\sigma = 0.6$

note: $\mu \pm 2\sigma = 1.2 \pm 2(0.6)$

$=$ from 0 to 2.4



all of the histogram falls within $\mu \pm 2\sigma$