Section 5.2: The Binomial Probability Distribution

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1:34 PM

binomial experiments:

1. have n identical trials

2. have only <u>two</u> possible outcomes

   Yes / No   Up / Down

   On / Off   Pass / Fail

   → we call one of these outcomes a success and the other one a failure

3. the probability of success is equal to p and remains the same from trial to trial

$$P(\text{success}) = p$$
$$P(\text{failure}) = 1 - p = q$$

4. the trials are independent

5. we are interested in x, the number of successes observed during the n trials

$$x = 0, 1, 2, \dots n$$

   note: x is <u>bounded</u> with max & min values

example: under what conditions is the following a binomial experiment?

(1) 3 red and 4 black marbles are put into a bag
(2) the experimenter mixes them thoroughly and then draws one and records the colour
(3) step 2 is repeated for a total of 4 trials

if you do not replace the marbles between trials, then the probabilities of red vs black change depending on the previous result and trials are not independent

∴ to be a binomial experiment, must be done with replacement

note: if there were 30 million red marbles and 40 million black marbles, the probabilities of red vs black would change so little without replacement that the experiment is approximately binomial

so there's a rule for selecting without replacement:

if $\frac{n}{N} \geq \frac{1}{20}$ (or 0.05),

then cannot approximate with binomial

where n = number of trials
N = population size

in other words, if

population > 20 (sample size),

use binomial

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how do we calculate probability distribution?

suppose we have 3 trials, and probability of success is p and the probability of failure is q

sample space:

	# successes	probability of event
SSS	3	ppp
SSF	2	ppq
SFS	2	pqp
SFF	1	pqq
FSS	2	etc
FSF	1	
FFS	1	
FFF	0	

1 way to get 3 successes: so $P(3) = 1 \cdot p^3$

3 ways to get 2 successes: $P(2) = 3 \cdot p^2 q$

3 ways to get 1 success: $P(1) = 3 \cdot p \cdot q^2$

1 way to get 0 successes: $P(0) = 1 \cdot q^3$

generalizing:

$$p(x = k) = \underbrace{{}_nC_k}_{\text{combination!}} \; p^k q^{n-k}$$