Section 5.4: cont'd

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1:31 PM

hypergeometric distribution:

population with a total number of N containing M successes and N - M failures

choosing without replacement

the probability of exactly k successes in a random sample of size n is

choosing k out of M successes

choosing n-k out of N-M failures

$$P(x=k) = \frac{{}_{M}C_{k} \quad {}_{N-M}C_{n-k}}{{}_{N}C_{n}}$$

with mean:

$$\mu = n\left(\frac{M}{N}\right)$$

and variance:

$$\sigma^2 = n\left(\frac{M}{N}\right)\left(\frac{N-M}{N}\right)\left(\frac{N-n}{N-1}\right)$$

note: this distribution variance looks like the binomial but it's corrected for a finite population

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example:

A pack of computer chips (20 in total) has 5 defective chips. If three chips are randomly sampled,

a) what is the probability distribution for x, the number of defective chips in the sample?

hypergeometric

$N = 20$
$n = 3$
$M = 5$
$N - M = 15$
$k = x$

and $p(x=k) = \dfrac{{}_{M}C_{k} \; {}_{N-M}C_{n-k}}{{}_{N}C_{n}}$

$p(x) = \dfrac{{}_{5}C_{x} \; {}_{15}C_{3-x}}{{}_{20}C_{3}}$

your answer

$p(0) = \dfrac{{}_{5}C_{0} \; {}_{15}C_{3}}{{}_{20}C_{3}}$

$= \dfrac{91}{228} \approx 0.399$

$p(1) = 0.46$
$p(2) = 0.13$
$p(3) = 0.0088$

note: sum of probabilities

$= 1$ (good!)

b) what is the mean and standard deviation for the number of defective chips found in your sample?

$$\mu = n\left(\frac{M}{N}\right) = 3\left(\frac{5}{20}\right) = \boxed{\frac{3}{4} = 0.75}$$

$$\sigma^2 = n\left(\frac{M}{N}\right)\left(\frac{N-M}{N}\right)\left(\frac{N-n}{N-1}\right)$$

$$= 3\left(\frac{5}{20}\right)\left(\frac{15}{20}\right)\left(\frac{17}{19}\right)$$

$$\approx 0.503$$

$$\sigma \approx 0.709$$

$$\approx 0.71$$

c) If this batch of computer chips is rejected if one or more defectives are found, what's the probability of the batch being rejected?

$$p(x \geq 1) = 1 - p(x=0)$$

$$= 1 - 0.399$$

$$= 0.601$$