Section 6.1: Probability Distributions for Continuous Random Variables

Monday, May 27, 2013
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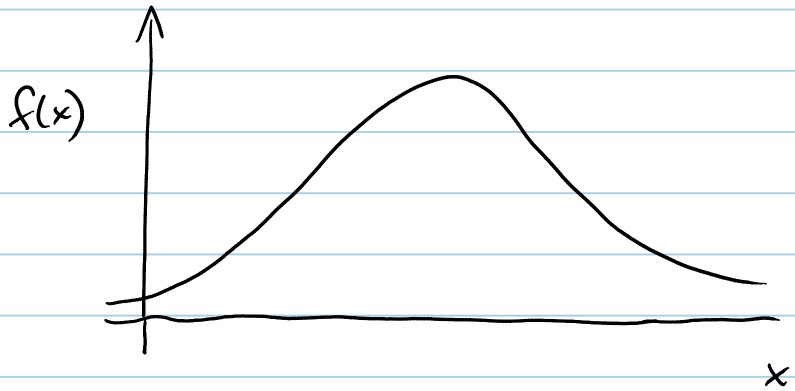
continuous random variables - have an infinite number of possible values

→ what, then, is the probability of getting a particular value?

it's identically zero (!)

how to deal?



you represent the data by a nice continuous curve where

f(x) = probability density function

properties:

- the area under the curve equals one

$$\int^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty} f(x)\,dx = 1$$

- the probability of x falling between a and b is:

$$P(a < x < b) = \int_a^b f(x)\,dx$$

- this is just the area under the curve from a to b

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note: $P(x = a) = 0$

$\therefore \quad P(x \geq a) = P(x > a)$

note: not true for discrete variables!

So, what $f(x)$ do you use? which function do you pick?

answer: the one that best models the situation (if such a model exists)

or

the one that best fits your data

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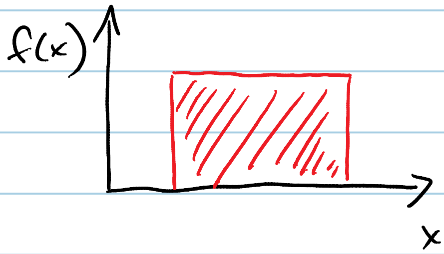
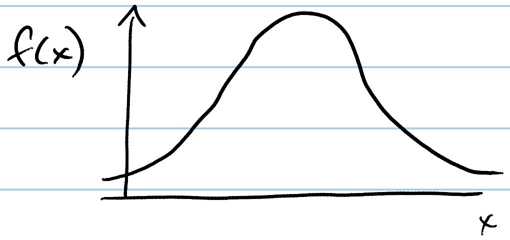
we'll look at a few of:

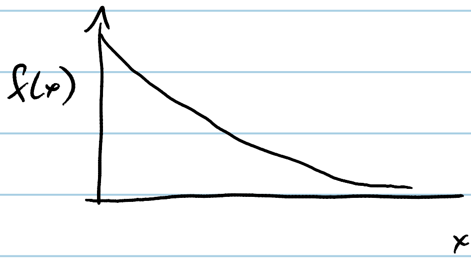


uniform distribution



normal distribution
(Bell curve)

others:



exponential distribution

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so, how do you find the mean?

$$\mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

and the variance?

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx \quad \leftarrow \text{annoying formal definition}$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2 \quad \leftarrow \text{useful "calculating formula"}$$

example: let x denote the amount of time for which a book is checked out of the library if it is on two-hour reserve

suppose that

$$f(x) = \begin{cases} 0.5x & \text{for } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a) Calculate the probability that $x \le 1$ hour.

method #1:

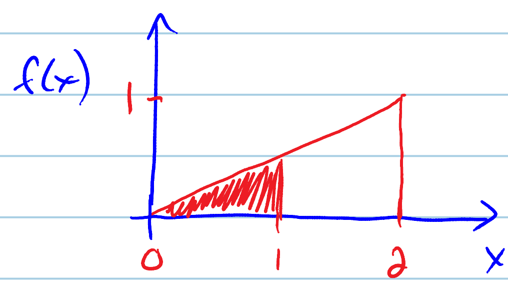
$$P(a < x < b) = \int_a^b f(x)\,dx$$

$$P(0 < x < 1) = \int_0^1 0.5x\,dx$$

$$= \frac{x^2}{4}\Big|_0^1$$

$$= \frac{1}{4} \quad \text{or} \quad 25\%$$

method #2:



$$P(0 < x < 1) = \text{shaded area}$$

$$= \tfrac{1}{2} b h$$

$$= \tfrac{1}{2} \cdot 1 \cdot \tfrac{1}{2} = \tfrac{1}{4}$$

b) calculate the mean value of x

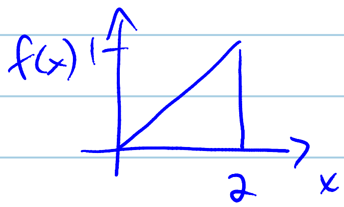
$$\mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{0}^{2} x\,(0.5\,x)\, dx$$

$$= \int_{0}^{2} \frac{1}{2} x^2\, dx$$

$$= \frac{x^3}{6}\Big|_0^2 = \frac{8}{6} = \boxed{\frac{4}{3}}$$

c) verify that the area under the curve is equal to one



$$\text{area} = \tfrac{1}{2} bh = \tfrac{1}{2} \cdot 2 \cdot 1 = 1$$