

Section 8.2: Large-Sample Confidence Intervals

Wednesday, June 05, 2013
11:56 AM

a confidence interval is the same idea as a point estimate, but you give the full interval (range of values) as your answer

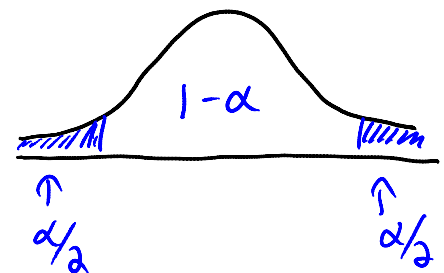
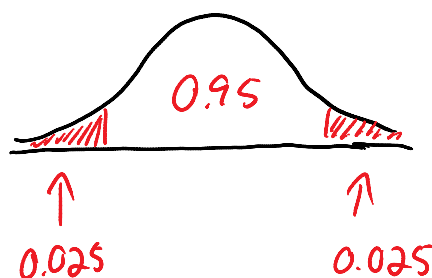
→ one difference - in general, you must state the confidence level you are using, rather than just assuming 95% confidence

confidence coefficient: $1 - \alpha$

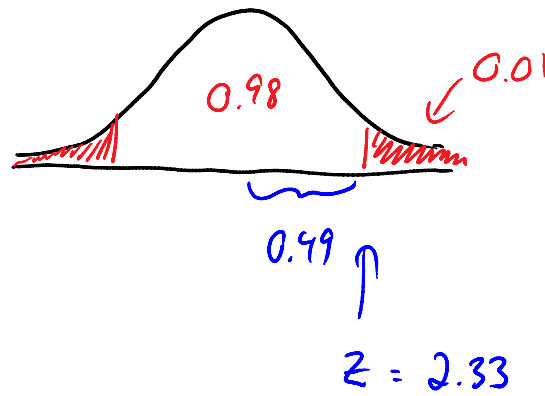
- $(1 - \alpha)$ is the probability that a confidence interval will contain the true value you are trying to estimate

(α itself is called the significance level)

- for 95% confidence, $1 - \alpha = 0.95$



- for 98% confidence, $1 - \alpha = 0.98$



Confidence Coefficient $1 - \alpha$	$Z_{\alpha/2}$
0.90	1.645
0.95	1.96
0.98	2.33
0.99	2.58

So then your confidence interval for the population mean is just

$$\mu = \bar{x} \pm \text{MOE}$$

margin of error
↓

$$= \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

provided that your sample size is large, $n \geq 30$.

note: for large sample sizes, when σ

unknown, okay to replace it
with S

\uparrow
"ess", std dev of sample

example: Recall that for Douglas fir trees in North America, a sample of 75 plots yielded a sample mean of 85 cm and a standard deviation of 12 cm for the diameter of those trees. Calculate the

- 90% confidence interval
- 95% confidence interval
- 99% confidence interval

for the mean diameter of Douglas firs in North America.

$$\begin{aligned} \text{a)} \quad \mu &= \bar{x} \pm Z_{\alpha/2} \frac{S}{\sqrt{n}} \\ &= 85 \pm 1.645 \frac{(12)}{\sqrt{75}} \\ &= 85 \pm 2.28 \\ &= 85 \pm 2 \text{ cm} \end{aligned}$$

The 90% CI is between 83 and 87 cm.

b) change 1.645 \rightarrow 1.96

$$\mu = 85 \pm 3 \text{ cm}$$

The 95% CI is between 82 and 88 cm.

c) change 1.645 \rightarrow 2.58

The 99% CI is between 81 and 89 cm.