

Section 8.3: Small-sample situation:

Monday, June 10, 2013
1:33 PM

Student's t distribution

What if your sample size is small?

- either the availability of data is restricted

(studying large-magnitude earthquakes)

- or practical considerations limit the amount of data you are willing to take

(cost of radiocarbon dating \$10,000 per sample)

(length of time per measurement)

When can you use the normal distribution?

$$\left. \begin{array}{l} \bar{x} \\ z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \end{array} \right\} \text{ normally distributed if}$$

either (1) original population you are sampling from is normal for any sample size

usual scenario (2) original population is not normal but sample size ≥ 30

conclusion: if n is small ($n < 30$)
and the distribution of the population
is unknown
→ cannot use normal!

Student's t -distribution

- used when sample size is small ($n < 30$)
- distribution shape depends on

$$\begin{aligned}df &= \text{degrees of freedom} \\ &= n - 1\end{aligned}$$

where $n =$ sample size

- shape of the distribution

- mound-shaped
- centred at $t = 0$

- but has more area in "tails"
than the normal distribution

- and as $n \rightarrow \infty$, the t -distribution
approaches the normal distribution

90% confidence interval with sample size of S : ↙ five



$$\begin{aligned}df &= n - 1 \\ &= 4\end{aligned}$$



$$df = n - 1 \\ = 4$$

$$t_{\alpha/2} = t_{0.05} = 2.132$$

So then the confidence interval is calculated by

$$\mu = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

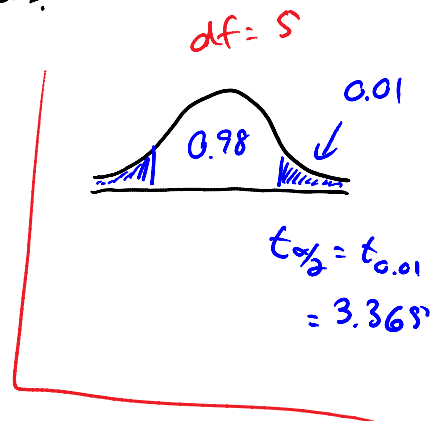
← standard deviation

Example: The duration for a random sample of 6 earthquakes in Southern California has been measured to be:

1.1, 0.9, 1.5, 0.7, 1.4, and 1.3 minutes

This sample has mean of 1.15 minutes with standard deviation 0.31 minutes. Give a 98% confidence interval for the duration of Southern California earthquakes.

$$\begin{aligned} \mu &= \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \\ &= 1.15 \pm \frac{3.365 (0.31)}{\sqrt{6}} \\ &= 1.15 \pm 0.425864 \end{aligned}$$



$$= 1.15 \pm 0.43 \text{ minutes}$$

The 98% confidence interval for duration of Southern CA earthquakes is 0.7 to 1.6 minutes.