Review:

Wednesday, June 12, 2013
11:37 AM

Solve the following DE:

$$\frac{dy}{dx} + 2xy^2 = 0$$

$$\frac{dy}{dx} = -2xy^2$$

$$\int \frac{dy}{y^2} = \int -2x\,dx$$

$$\int y^{-2}\,dy = -x^2 + C$$

$$\frac{y^{-1}}{-1} = -x^2 + C$$

$$\left.\begin{aligned} -y^{-1} &= -x^2 + C \\ y^{-1} &= x^2 + C^* \end{aligned}\right\} \text{acceptable (implicit sol'n)}$$

note: solve explicitly means solve for dependent variable – in this case, y

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Solve $$x\frac{dy}{dx} + 2y = 3x^2$$

linear 1st order

$$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = 3x$$

$$+\left(\frac{2}{x}\right)y = 3x$$

IF $e^{\int P(x)dx} = e^{\int \frac{2}{x}dx}$

$= e^{2\ln x}$

$= e^{\ln x^2}$

$= x^2$

$$x^2\frac{dy}{dx} + 2xy = 3x^3$$

$$\frac{d}{dx}(x^2y) = 3x^3$$

$$\int d(x^2y) = \int 3x^3dx$$

$$x^2y = \frac{3}{4}x^4 + C$$

now, write an explicit solution:

$$y = \frac{3}{4}x^2 + \frac{C}{x^2}$$

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solve: $y'' - 6y' + 8y = 24x^2$

complementary solution: aux eqn: $m^2 - 6m + 8 = 0$

$(m-2)(m-4) = 0$

$m = 2, 4$

$$y_c = C_1 e^{2x} + C_2 e^{4x}$$

particular solution:

$$y_p = Ax^2 + Bx + C$$

no bad case!

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

plug into DE:

$$y'' - 6y' + 8y = 24x^2$$

$$2A - 6(2Ax + B) + 8(Ax^2 + Bx + C) = 24x^2$$

$$2A - 12Ax - 6B + 8Ax^2 + 8Bx + 8C = 24x^2$$

$$8Ax^2 + x(-12A + 8B) + (2A - 6B + 8C) = 24x^2$$

then $8A = 24$

$A = 3$

$-12A + 8B = 0$

$-36 + 8B = 0$

$B = 36/8 = 9/2$

$2A - 6B + 8C = 0$

$6 - 27 + 8C = 0$

$8C = 21$

$C = 21/8$

$$y_p = 3x^2 + \frac{9}{2}x + \frac{21}{8}$$

$$\boxed{y = y_c + y_p = c_1 e^{2x} + c_2 e^{4x} + 3x^2 + \frac{9}{2}x + \frac{21}{8}}$$