Section 7.2: cont'd:

Monday, June 03, 2013
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Assignment #4 due on

Tuesday, June 11^{th}

Central Limit Theorem:

if random samples of n observations are drawn from a non-normal population with finite mean μ and standard deviation σ,

when n is large, the distribution of the sample mean $\bar{x}$ is approximately normally distributed with

mean μ

standard deviation $\frac{\sigma}{\sqrt{n}}$

and this approximation gets more accurate as n increases

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example: suppose that at a large university, there are 12 sections of intro calculus running concurrently

→ consider the probability of an individual student getting >90% on a test

is this probability for an individual the same or different than the probability that one of the sections will have a section average >90% ?

- different! it's far more likely for an individual to get 90% than for an entire group to do so

and the larger the group, the less likely it is

what the central limit theorem says for this example is that :

if on the test, the average for an individual is 72 out of 100 with a standard deviation of 8 out of 100,

then the class average for a group of 16 students would be still 72 out of 100

but the standard deviation is now $\frac{8}{\sqrt{n}} = \frac{8}{\sqrt{16}} = 2$

summary: the mean of a group will be normally distributed with

mean μ

std dev $\frac{\sigma}{\sqrt{n}}$

the sum of a group has

mean $n\mu$

std dev $\sigma\sqrt{n}$

} previous result multiplied by n

So the spread of the sample means is much less than the spread of the sampled population

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example: The weight of luggage checked by airline passengers is a random variable with a mean of 50 lbs and a standard deviation of 30 lbs. The total baggage limit for 100 randomly selected passengers is 5750 lbs.

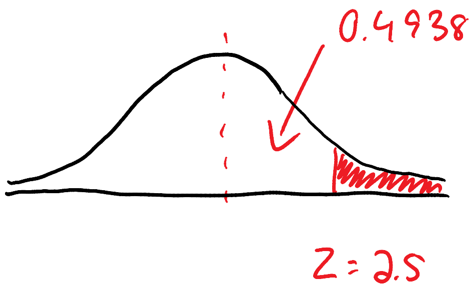
What is the approximate probability that the baggage limit will be exceeded?

the sum of the weights of luggage will be approximately normally distributed because $n = 100$ is large (guidelines later)

$$\mu_{\text{total weight of luggage}} = n \, \mu_{\text{individual}}$$
$$= 100 \cdot 50$$
$$= 5000$$

$$\sigma_{\text{total weight}} = \sigma \sqrt{n}$$
$$= 30 \sqrt{100}$$
$$= 300$$

$$Z = \frac{x - \mu}{\sigma} = \frac{5750 - 5000}{300}$$
$$= 2.5$$



$$P(Z > 2.5) = 0.5 - 0.4938$$
$$= 0.0062$$

There is 0.6 % chance that the baggage limit will be exceeded.

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the central limit theorem talks about " when the sample size is <u>large</u> "

How large?

- if the sampled population has a normal distribution, then $\bar{x}$ will be normally distributed, no matter how big or small n is

(n: sample size)

- if the sampled population has a roughly symmetrical distribution, then the sampling distribution becomes approximately normal for relatively small values of n

so if pop is symmetrical, samp. dist. may be normal for $n \geq 5$
(but doesn't have to be!)

- if sampled population is skewed, need at least 30 samples before their means/sums become approximately normally distributed

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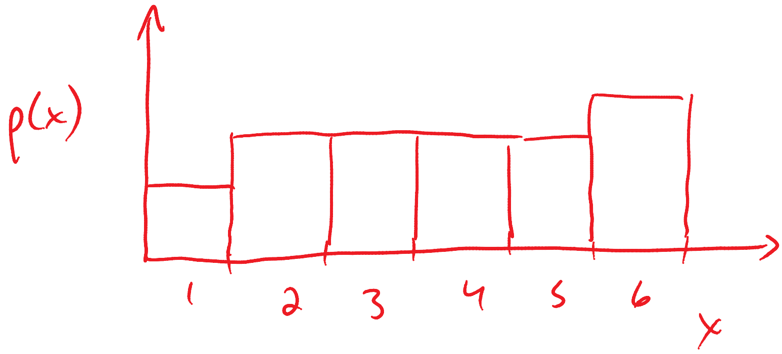
example: A handful of six-sided dice are loaded - the probability of rolling a 6 is significantly higher (and therefore the probability of rolling a 1 significantly less) than the other numbers.

a) Sketch the probability distribution of the number rolled on a single die

b) sketch the probability distribution of the sum of fifty such dice

a)



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