Math 189 – Test #2

May 19, 2015 **Name:** Solution Set
Instructor: Patricia Wrean

Total: 25 points

1. State the form of the particular solution y_p for the following. Leave your answer with undetermined coefficients. (This means "Write down your initial guess for y_p but don't bother to solve for the constants.") Please note that the complementary solution for the homogeneous equation is $y_c = C_1 + C_2e^{6x}$. (3 points)

   a) $y'' - 6y' = 1 - 5e^{-x}$ $y_p = Ax + \underbrace{Be^{-x}}_{-Be^{-x} \text{ also acceptable}}$ 2

   b) $y'' - 6y' = -3\cos x$ $y_p = A\cos x + B\sin x$ 1

a) is "bad case": $y_p = A + Be^{-x}$

but this is a "like term" to C_1, so mult by x

$= Ax + Be^{-x}$

-1 if not notice "bad case"

2. Consider the following variables concerning a car. (2 points)

   a) the year it was built quant - discrete

   b) the name of the manufacturer qual

   c) the gas mileage quant - cont

   d) the colour qual

   Which of these are quantitative? a, c 1

   From the quantitive data, which are continuous? c 1

3. According to the Vancouver Canucks' website (www.canucks.com), the number of goals scored by their top ten scorers in a past year are as follows: (5 points)

36, 24, 23, 20, 18, 14, 12, 12, 12, 11.

a) State the mean, median, and range of this data set.

okay to report as 18 → mean: 18.2 (1)

median = average of 18 + 14 median: 16 (1)

range = max − min range: 25 (1)

= 36 − 11 = 25

(−½) if wrote 11 − 36

b) Suppose that in the next game or games, the highest and lowest numbers of goals (36 and 11) each increased by two while all of the other data points stayed the same. What would happen to the median and the range? Be as specific as you can!

⇒ 38, 24, 23, 20, 18, 14, 13, 12, 12, 12

median still same (1)

range now 38 − 12 = 26

so range increases by one (1)

4. Consider the following sets of data. Without calculating any values, indicate which set will have the higher standard deviation (or will they be the same?). (2 points)

a) Set 1: 1, 3, 5, 7, 9 Set 2: 1, 2, 3, 4, 5

all closer to mean

b) Set 1: 1, 3, 5, 7, 9 same Set 2: 11, 13, 15, 17, 19

c) Set 1: 1, 4, 5, 6, 9 Set 2: 3, 4, 5, 6, 7

1 + 9 further away

(½) each

d) Set 1: 22, 23, 24, 25, 26 Set 2: 15, 20, 25, 30, 35

more spread

5. A set of data has a mean of 75 and a standard deviation of 5. After looking at the relative frequency histogram, you know that this is a more-or-less symmetric, mound-shaped distribution. (~~3~~ 4 points)

a) What can you estimate about the proportion of measurements that fall between 65 and 85?

65 to 85 equals $\bar{x} \pm 2s$ (or $\mu \pm 2\sigma$) 1

by the Empirical rule, ~95 % of data lie within 2 std dev of mean 1

b) What can you say with certainty about the proportion of measurements that fall between 65 and 85?

by Tchebysheff, ≥ 75 % of data lie within 2 std dev of mean 1

c) Tchebysheff says that at least 50% of the data will fall between the values x and y. Calculate those values.

$$1 - \frac{1}{k^2} = 0.5$$ 1

$$0.5 = \frac{1}{k^2}$$

$$\frac{1}{2} = \frac{1}{k^2}$$

$$k = \pm\sqrt{2}$$

$$= +\sqrt{2}$$

≥ 50% of data lie within $\bar{x} \pm ks$

$$= 75 \pm \sqrt{2} \cdot 5$$

$$= 75 \pm 7.07$$

$$\boxed{= 68 \text{ to } 82}$$

6. Solve: $y'' + 2y' - 3y = 2 + 12e^{-3x}$. (5 points)

complementary soln:

$$m^2 + 2m - 3 = 0$$

$$(m+3)(m-1) = 0$$

$$m = 1, -3$$

$$y_c = c_1 e^{x} + c_2 e^{-3x} \quad 1$$

particular solution:

$$y_p = A + \cancel{Be^{-3x}} \quad \text{bad case}$$

$$= A + Bxe^{-3x} \quad 1$$

bad case omitted so B's cancel: -2

$$y_p' = Be^{-3x} - 3Bxe^{-3x}$$

$$y_p'' = -3Be^{-3x} - 3Be^{-3x} + 9Bxe^{-3x}$$

$$= -6Be^{-3y} + 9Bxe^{-3x} \quad 1$$

sub back into DE:

$$y'' + 2y' - 3y = 2 + 12e^{-3x}$$

$$-6Be^{-3x} + 9Bxe^{-3x} + 2\left(Be^{-3x} - 3Bxe^{-3x}\right) - 3\left(A + Bxe^{3x}\right) = 2 + 12e^{-3x}$$

$$-6Be^{-3x} + \cancel{9Bxe^{-3x}} + 2Be^{-3x} - \cancel{6Bxe^{-3x}} - 3A - \cancel{3Bxe^{3x}} = 2 + 12e^{-3x}$$

$$-4Be^{-3x} - 3A = 2 + 12e^{-3x} \quad 1$$

$$-3A = 2 \quad \text{and} \quad -4B = 12$$

$$A = -2/3 \qquad B = -3$$

so $y_p = -2/3 - 3xe^{-3x}$ 1

and $\boxed{y = y_c + y_p = c_1 e^{x} + c_2 e^{-3x} - 2/3 - 3xe^{-3x}}$

7. A 0.50 kg mass is suspended from a spring with spring constant $k = 12.5$ N/m. The mass moves up and down according to the following differential equation, where y is the height of the block (in cm) above its equilibrium position.

I meant to use m here - any student who noticed the cm and tried to deal with it got (+½) bonus point

$$m\frac{d^2y}{dt^2} + ky = 0$$

A student taps the mass so that it starts at the equilibrium position with an initial speed of 1.5 m/s downward. Find the distance y as a function of the time elapsed after the initial tap. (~~6~~ 4 points)

DE becomes $0.50\,\ddot{y} + 12.5\,y = 0$

aux eqn: $0.50\,n^2 + 12.5 = 0$

$n^2 = -25$

$n = \pm 5i$ (well, $5.0i$, actually)

$= \alpha \pm \beta i$ where $\alpha = 0$, $\beta = 5$ (1)

$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t)$

$= c_1 \cos 5t + c_2 \sin 5t$ (1)

initial conditions: at $t = 0$, $y = 0$ and $y' = -1.5$ m/s

$0 = c_1 \cdot 1 + c_2 \cdot 0$ so $c_1 = 0$

$y = c_2 \sin 5t$

$y' = 5c_2 \cos 5t$ (½)

at $t = 0$ $-1.5 = 5c_2 \cdot 1$

$c_2 = -0.30$

$\boxed{y = -0.30 \sin 5t}$ (1½)