Math 189 – Test #3

June 11, 2015
Instructor: Patricia Wrean

Name: Solution Set

Total: 25 points

1. A jar contains four coins: a dime, a quarter, a loonie, and a toonie. Three coins are randomly selected from the jar. (3 points)

   a) List the simple events in the sample space.

   dqℓ
   dqt
   qℓt
   qℓt

   4 events

   (could also write "no toonie" "no loonie" "no quarter" "no dime")

   b) What is the probability that the selection will contain both the loonie and the toonie? Show enough work that I can see the method you are using.

   $$P(\ell t) = \frac{n(\ell t)}{n} = \frac{2}{4} = \boxed{\tfrac{1}{2} \text{ or } 50\%}$$

2. A certain electronic lock has ten buttons on its face, numbered from 0 to 9. (4 points)

   a) If you open the lock by pressing any three buttons one at a time in a definite sequence, how many different ways could you try to open the lock?

   $\underline{10}\ \underline{10}\ \underline{10} = 1000$

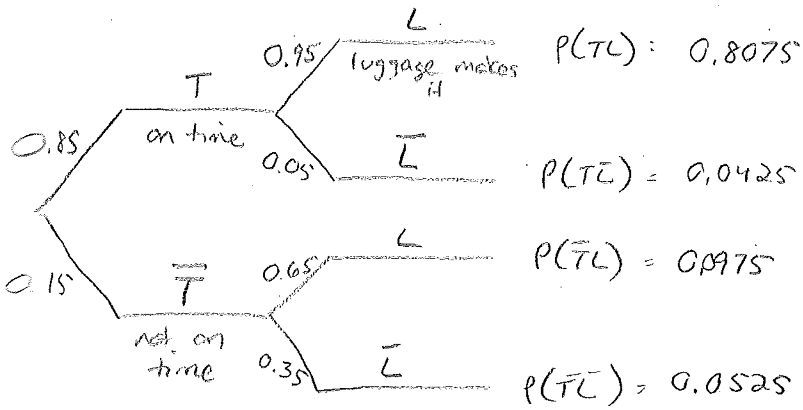
   b) If you open the lock by pressing three different buttons all at the same time, how many different ways could you try to open the lock?

   combination

   $${}_nC_r = {}_{10}C_3 = 120$$

3. Leah is flying from Victoria to Toronto with a stop in Vancouver. The probability that her first flight leaves on time is 85%. If the flight is on time, the probability that her luggage will make the connecting flight in Vancouver is 95%, but if her first flight is delayed, the probability that the luggage will make it is only 65%. (5 points)

a) What is the probability that her luggage makes the connecting flight?



3

$$P(L) = P(TL) + P(\bar{T}L)$$
$$= 0.8075 + 0.0975$$
$$= 0.905 = \boxed{90.5\%}$$

if give $P(T|L)$ instead

-2

b) Are "the first flight leaving on time" and "the luggage makes the connection" independent events? Explain, including ~~calculations~~ values of appropriate probabilities.

if independent, $P(L) = P(L|T)$

$P(L) = 90.5\%$

$P(L|T) = 95\%$

$\therefore$ not independent

½ correct conclusion

1½ explanation

2

or if independent, $P(T) = P(T|L)$

$P(T) = \boxed{85\%}$

$P(T|L) = \frac{P(LT)}{P(L)} = \frac{0.8075}{0.905} = 0.8923 = \boxed{89\%}$

$\therefore$ same conclusion

4. Thrifty's is considering opening a new store at a certain location in Saanich. They conduct a survey, which estimates that this location has a 70% chance of success with an annual profit of \$150,000 if it is successful and a \$60,000 loss otherwise. If the company policy is to only open a new store if the expectation value of the profit exceeds \$100,000, should they open this new store? (3 points)

x	$p(x)$
150,000	0.7
-60,000	0.3

1

$E(x) = \sum x\, p(x)$

$= 150{,}000(0.7) + (-60{,}000)(0.3)$

$= \$87{,}000$, which is less than the \$100,000 threshold

1

$\boxed{\text{Thrifty's should not open this new store.}}$

5. The average number of dandelions in Pat's front lawn is six. (3 points)

a) Let x be the number of dandelions found in her lawn today. What is the name of the probability distribution that would best describe x?

Poisson (number of events in given space/time)

b) Using the distribution you've chosen, calculate the probability that there is more than one dandelion in Pat's front lawn today.

$P(x=k) = \dfrac{\mu^k e^{-\mu}}{k!}$ so

$P(x>1) = 1 - P(x=0) - P(x=1)$

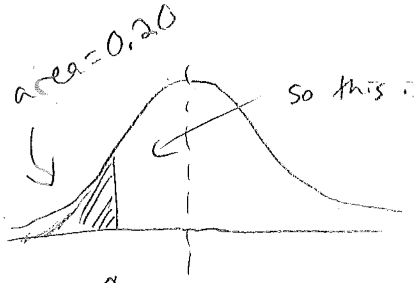
$= 1 - \dfrac{6^0 e^{-6}}{0!} - \dfrac{6^1 e^{-6}}{1!}$

$= 0.982649$

$= \boxed{98\%}$

6. The mayor of Victoria was informed that household water usage is a normally distributed random variable with mean of 25 gallons/day and a standard deviation of 6 gallons/day. (4 points)

   a) If the mayor wants to give a tax rebate to the lowest 20% of water users, what should the gallons/day cutoff be?



2

$z = -0.84$ from normal table

$$z = \frac{x-\mu}{\sigma}$$

$$x = \mu + z\sigma$$

$$= 25 + (-0.84)(6)$$

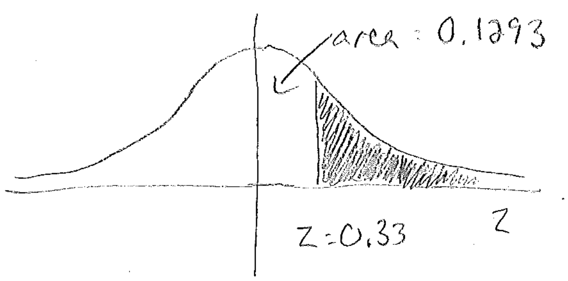
$$= 19.96$$

$$\approx 20$$

$\boxed{\text{The cutoff should be 20 gallons/day.}}$

   b) Calculate the probability that a randomly-chosen household will use more than 27 gallons per day.

$$z = \frac{x-\mu}{\sigma} = \frac{27-25}{6} = 0.33$$



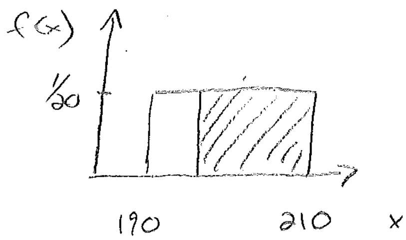
$$P(z > 0.33) = 0.5 - 0.1293$$

$$= 0.3707$$

There is a 37% probability that a randomly-chosen household will exceed 27 gallons per day.

7. A soft-drink machine is regulated so that it discharges an amount of liquid which is a uniform random variable with values between 190 and 210 mL. (3 points)

a) Calculate the fraction of drinks dispensed that will have a volume greater than 195 mL.



$P(x > 195) = \boxed{75\%}$ (area of shaded region = 3/4 of total area)

b) Calculate the mean and standard deviation of the volume of liquid this machine dispenses.

$\boxed{\mu = 200 \text{ mL}}$ (centre of the rectangle)

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= \int_{190}^{210} x^2 \cdot \frac{1}{20}\,dx - 200^2$$

$$= \frac{1}{60} x^3 \Big|_{190}^{210} - 200^2$$

$$= \frac{1}{60}(210^3 - 190^3) - 200^2$$

$$= 33.\bar{3}$$

so $\sigma \approx 5.7735$ mL

$\boxed{\sigma \approx 6 \text{ mL}}$