Application of Matrix Multiplication: Vector Rotation

In 2D: Rotating a vector by an angle θ can be done using multiplication by the matrix

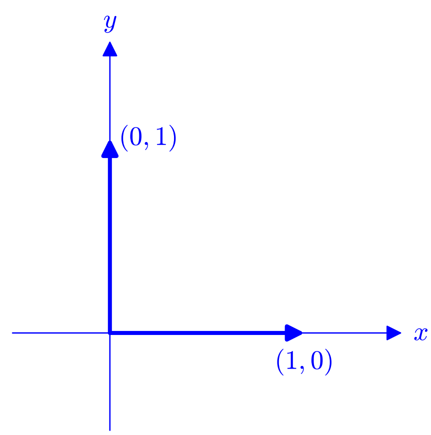
$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

For example, for $\theta = 90°$, the rotation matrix is:

$$R_{90°} = \begin{bmatrix} \cos 90° & -\sin 90° \\ \sin 90° & \cos 90° \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

To rotate the vector $\mathbf{v} = (1, 0)$ by $90°$:

$$R\mathbf{v} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$



Example: Rotate the vector $\mathbf{v} = (1, 1)$ by **(a)** $90°$ **(b)** $45°$.

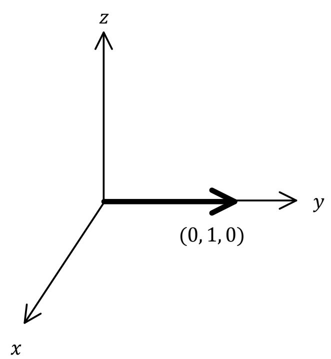
In 3D: Rotating a vector by an angle θ is axis-specific. To rotate around the x-axis means that the rotation is happening counterclockwise and parallel to the yz-plane. The rotation matrices for rotation around each axis are:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

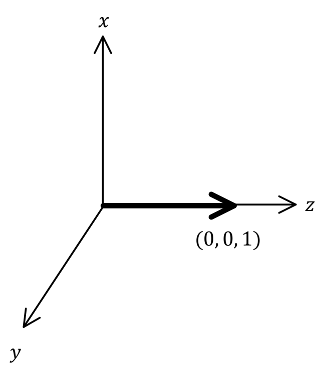
$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example 1: Rotate the vector $\mathbf{v} = (0, 1, 0)$ by 90° around the x-axis.



Example 2: Rotate the vector $\mathbf{v} = (0, 0, 1)$ by 90° around the y-axis.



Example 3: Rotate the vector $\mathbf{v} = (1, 0, 0)$ by 90° around the z-axis.