Supplementary Questions for 23.6 and 23.7

Differentiate the following:

1. $f(x) = \dfrac{3x+8}{x^2+4x+2}$ u v

$$f'(x) = \frac{vu' - uv'}{v^2} = \frac{(x^2+4x+2)(3) - (3x+8)(2x+4)}{(x^2+4x+2)^2}$$

$$= \frac{3x^2+12x+6 - (6x^2+28x+32)}{(x^2+4x+2)^2}$$

$$= \frac{-3x^2-16x-26}{(x^2+4x+2)^2}$$

2. $y = \dfrac{4}{\sqrt[5]{x^2}} = 4x^{-2/5}$

$$y' = 4\left(-\frac{2}{5}\right)x^{-7/5} = \frac{-8}{5x^{7/5}} = \frac{-8}{5\sqrt[5]{x^7}}$$

3. $f(x) = \dfrac{4}{x^2} - \dfrac{3}{x^5} = 4x^{-2} - 3x^{-5}$

$$f'(x) = -8x^{-3} + 15x^{-6} = \frac{-8}{x^3} + \frac{15}{x^6}$$

4. $s(t) = 3(8t^2-7)^6$

$$s'(t) = 3 \cdot 6(8t^2-7)^5(16t) = 288t(8t^2-7)^5$$

5. $y = \dfrac{\pi^3}{\sqrt{1-3x}}$ $= \pi^3 (1-3x)^{-1/2}$

$$y' = \pi^3 \left(-\tfrac{1}{2}\right)(1-3x)^{-3/2}(-3) = \frac{3\pi^3}{2(1-3x)^{3/2}}$$

6. $y = 9\sqrt[3]{4x^6+2}$ $= 9(4x^6+2)^{1/3}$

$$y' = 9\left(\tfrac{1}{3}\right)(4x^6+2)^{-2/3}(24x^5)$$

$$= \frac{72x^5}{(4x^6+2)^{2/3}}$$

7. $f(x) = \dfrac{2x^2}{\sqrt[5]{1+4x}}$ $= \underbrace{2x^2}_{u}\underbrace{(1+4x)^{-1/5}}_{v}$

$$f'(x) = uv' + vu' = 2x^2\left(-\tfrac{1}{5}\right)(1+4x)^{-6/5}(4) + (1+4x)^{-1/5}(4x)$$

$$= \frac{-8x^2}{5(1+4x)^{6/5}} + \frac{4x}{(1+4x)^{1/5}} \cdot \frac{5(1+4x)}{5(1+4x)}$$

$$= \frac{72x^2+20x}{5(1+4x)^{6/5}}$$

8. $f(x) = x^3\sqrt{4x+5}$

$= \underbrace{x^3}_{u}\underbrace{(4x+5)^{1/2}}_{v}$

$$f'(x) = uv' + vu'$$

$$= x^3\left(\tfrac{1}{2}\right)(4x+5)^{-1/2}(4) + (4x+5)^{1/2}(3x^2)$$

$$= \frac{2x^3}{(4x+5)^{1/2}} + 3x^2(4x+5)^{1/2} \cdot \frac{(4x+5)^{1/2}}{(4x+5)^{1/2}}$$

$$= \frac{2x^3 + 3x^2(4x+5)}{(4x+5)^{1/2}} = \frac{14x^3+15x^2}{(4x+5)^{1/2}}$$