

Optimization Review

1. A farmer wishes to fence a rectangular pasture for his herd of cows. One side of the pasture, which is to be adjacent to a river, requires no fencing. The total area of the pasture must be $125,000 \text{ m}^2$. What dimensions will require the least amount of fencing?

Solution: 500 m (along the river) by 250 m

2. A open box is to be made by cutting square pieces from the corners of a 3 cm by 8 cm sheet of cardboard and folding up the sides. Find the maximum volume that such a box can have.

Solution: 7.41 cm^3

3. The velocity of a particle (in ft/s) is given by $v(t) = 4t^3 - 9t^2 + 20$, where t is the time for which it has travelled. Find the minimum velocity of the particle for $t \geq 0$.

Solution: 13.25 ft/s

4. A piece of wire 100 cm long is going to be cut into 12 pieces and used to construct the skeleton (edges only) of a rectangular box. The base of the box is to be constructed so that the length is twice the width. The resulting frame is then covered with paper. What are the dimensions of the box with the largest

a) volume?

b) surface area?

Solution: a) $\frac{100}{9} \times \frac{50}{9} \times \frac{25}{3} \text{ cm}$ b) $\frac{75}{7} \times \frac{75}{14} \times \frac{125}{14} \text{ cm}$

5. A 1 m long piece of wire needs to be divided into two pieces which will form the shape of a circle and a square. Determine the length of each piece so that the sum of the areas of the circle and square is minimized.

Solution: 0.44 m for the circle and 0.56 m for the square

Related Rates Review

1. A balloon rises directly up at a rate of 8 ft/s from a point 60 feet from an observer. Find the rate of change of the angle of elevation when the balloon is 25 feet above the ground. Ignore the height of the observer.

Solution: 0.11 rad/s

2. The top of a ladder 5 m long is slipping down a vertical wall at the constant rate of 0.8 m/s. How fast is the angle between the ladder and the ground changing when the bottom of the ladder is 3 m from the wall?

Solution: 0.27 rad/s

3. The total inductance, L_T , of two inductors in parallel is given by

$$L_T = \frac{L_1 L_2}{L_1 + L_2}.$$

Suppose that L_1 is constant at 2 H and that L_2 is increasing at a rate of 0.5 H/s. Find the rate at which L_T is changing when $L_2 = 3$ H.

Solution: 0.08 H/s

4. Suppose that an inflating balloon is spherical in shape, and its radius is changing at the rate of 3 cm/s. At what rate is the volume changing when the radius is 10 cm? Recall that $V = \frac{4}{3}\pi r^3$.

Solution: 3769.91 cm³/s

5. A street light is at the top of a 9 ft tall pole. A woman 5 feet tall jogs away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of her shadow moving when she is 30 feet from the pole?

Solution: 11.25 ft/s