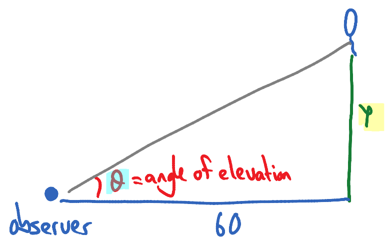
1. A balloon rises directly up at a rate of 8 ft/s from a point 60 feet from an observer. Find the rate of change of the angle of elevation when the balloon is 25 feet above the ground.



(ignore height of observer)

2 rates: given $\frac{dy}{dt} = 8 \ \frac{ft}{s}$

wanted $\left.\frac{d\theta}{dt}\right|_{y=25}$

so we need an equation relating only y and θ

$$\tan\theta = \frac{y}{60} \qquad \text{optional: } \theta = \tan^{-1}\left(\frac{y}{60}\right)$$

⋮

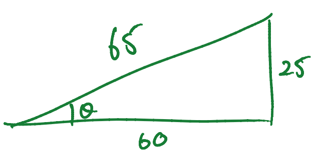
$$\frac{d}{dt}\left(\tan\theta\right) = \frac{d}{dt}\left(\frac{1}{60}y\right)$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{60}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{60\sec^2\theta}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{\cos^2\theta}{60}\frac{dy}{dt}$$

when $y = 25$:



$$\cos\theta = \frac{adj}{hyp} = \frac{60}{65}$$

$$\left.\frac{d\theta}{dt}\right|_{y=25} = \frac{\left(\frac{60}{65}\right)^2}{60}\cdot(8) \approx \boxed{0.11 \ \frac{rad}{s}}$$

2. The total inductance, L_T, of two inductors in parallel is given by

$$L_T = \frac{L_1 L_2}{L_1 + L_2}.$$

Suppose that L_1 is constant at 2 H and that L_2 is increasing at a rate of 0.5 H/s. Find the rate at which L_T is changing when $L_2 = 3$ H.

2 rates: given $\frac{dL_2}{dt} = 0.5 \frac{ft}{s}$

wanted $\left.\frac{dL_T}{dt}\right|_{L_2=3}$

so we need an equation relating only L_2 and L_T

$L_1 = 2$ so

$$L_T = \frac{2L_2}{2+L_2}$$

$$\frac{d}{dt}(L_T) = \frac{d}{dt}\left(\frac{2L_2}{2+L_2}\right)$$

$$\frac{dL_T}{dt} = \frac{(2+L_2)\cdot 2\frac{dL_2}{dt} - 2L_2\cdot\frac{dL_2}{dt}}{(2+L_2)^2}$$

$$\left.\frac{dL_T}{dt}\right|_{L_2=3} = \frac{(2+3)\cdot 2(0.5) - 2\cdot 3\cdot(0.5)}{(2+3)^2} = \boxed{0.08 \frac{ft}{s}}$$

3. Suppose that an inflating balloon is spherical in shape, and its radius is changing at the rate of 3 cm/s. At what rate is the volume changing when the radius is 10 cm? Recall that $V = \frac{4}{3}\pi r^3$.

2 rates: given $\frac{dr}{dt} = 3 \frac{cm}{s}$

wanted $\left.\frac{dV}{dt}\right|_{r=10}$

so we need an equation rating only r and V

$$V = \frac{4}{3}\pi r^3$$

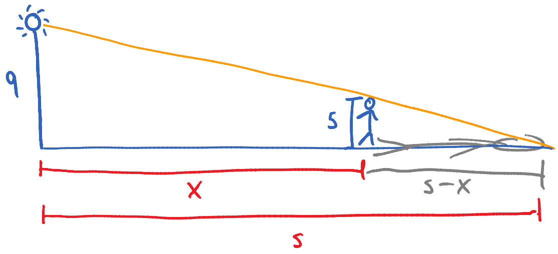
$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\left.\frac{dV}{dt}\right|_{r=10} = 4\pi(10)^2 \cdot (3) = 1200\pi \approx \boxed{3769.91 \ \frac{cm^3}{s}}$$

4. A street light is at the top of a 9 ft tall pole. A woman 5 feet tall jogs away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of her shadow moving when she is 30 feet from the pole?

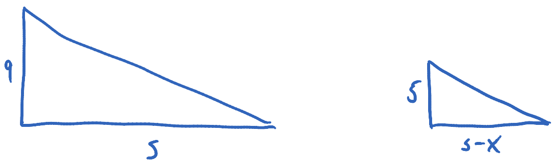


2 rates: given $\frac{dx}{dt} = 5 \ \frac{ft}{s}$

wanted $\left.\frac{ds}{dt}\right|_{x=30}$

so we need an equation relating only x and s

Similar triangles:



$$\frac{s}{9} = \frac{s-x}{5}$$

$$5s = 9(s-x)$$

$$5s = 9s - 9x$$

$$9x = 4s$$

$$s = \frac{9}{4}x$$

$$\frac{d}{dt}(s) = \frac{d}{dt}\left(\frac{9}{4}x\right)$$

$$\frac{ds}{dt} = \frac{9}{4}\frac{dx}{dt} = \frac{9}{4}(5) = \boxed{11.25 \ ft/s}$$